**Theorem** An Erlang$(\alpha, n)$ random variable multiplied by $2/\alpha$ is $\chi^2(2n)$ random variable.

**Proof** Let the random variable $X$ have the Erlang distribution with probability density function

$$f_X(x) = \frac{1}{\alpha^n (n-1)!} x^{n-1} e^{-x/\alpha} \qquad x > 0.$$

The transformation $Y = g(X) = 2X/\alpha$ is a 1–1 transformation from $\mathcal{X} = \{x \mid x > 0\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = \alpha Y/2$ and Jacobian

$$\frac{dX}{dY} = \alpha/2.$$

Therefore by the transformation technique, the probability density function of $Y$ is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{1}{\alpha^n (n-1)!} (\alpha y/2)^{n-1} e^{-(\alpha y/2)/\alpha} |\alpha/2| \\ &= \frac{1}{\alpha^n (n-1)!} \frac{\alpha^{n-1} y^{n-1}}{2^{n-1}} \frac{\alpha}{2} e^{-y/2} \\ &= \frac{1}{2^n \Gamma(n)} y^{n-1} e^{-y/2} \qquad y > 0, \end{aligned}$$

which is the probability density function of the $\chi^2(2n)$ distribution.

**APPL verification:** The APPL statements

```
X := ErlangRV(alpha, n);
g := [[x -> 2 * alpha * x], [0, infinity]];
Y := Transform(X, g);
```

yield the probability density function of a $\chi^2(2n)$ random variable. Note that APPL has the parameter $\alpha$ as $1/\alpha$ in our probability density function, so we use `2 * alpha * x` in the commands.