Erlang Distribution

The shorthand $X \sim \text{Erlang}(\alpha, n)$ is used to indicate that the random variable X has the Erlang distribution with scale parameter α and shape parameter n. An Erlang random variable X with scale parameter α and n stages has probability density function

$$f(x) = \frac{x^{n-1} e^{-x/\alpha}}{\alpha^n (n-1)!} \qquad x > 0.$$

The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - \sum_{i=0}^{n-1} \frac{e^{-x/\alpha} x^n}{\alpha^n n!} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \sum_{i=0}^{n-1} \frac{e^{-x/\alpha} x^n}{\alpha^n n!} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{x^{n-1} e^{-x/\alpha}}{\alpha^n (n-1)!} \sum_{i=0}^{n-1} \frac{\alpha^n n!}{e^{-x/\alpha} x^n} \qquad x > 0.$$

The cumulative hazard function is intractable.

The inverse distribution function of X is intractable.

There is no simple closed form of the median.

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = (1 - t\alpha)^{-n} \qquad t < 1/\alpha.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = (1 - it\alpha)^{-n} \qquad t < 1/\alpha.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = n\alpha \qquad V[X] = n\alpha^2 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2}{\sqrt{n}} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{6}{n}.$$