Theorem Random variates from the discrete Weibull(p, β) distribution can be generated in closed-form by inversion.

Proof The discrete Weibull(p, β) distribution has probability mass function

$$f(x) = (1-p)^{x^\beta} - (1-p)^{(x+1)^\beta} \qquad x = 0, 1, 2, \ldots,$$

for $0 < p < 1$ and $\beta > 0$. The cumulative distribution function is

$$F(x) = 1 - (1-p)^{(x+1)^\beta} \qquad x = 0, 1, 2, \ldots.$$

Equating the cumulative distribution function to u, where $0 < u < 1$, yields an inverse cumulative distribution function

$$F^{-1}(u) = \left\lceil \left(\frac{\ln(1-u)}{\ln(1-p)} \right)^{1/\beta} - 1 \right\rceil \qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the discrete Weibull(p, β) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \left\lceil (\ln(1-u)/\ln(1-p))^{1/\beta} - 1 \right\rceil$
return(X)

APPL failure: The APPL statements

```
X := [[1 - (1 - p) ^ ((x + 1) ^ beta)], [0, infinity], ["Discrete", "CDF"]];
IDF(X);
```

fail to produce the inverse distribution function of a discrete Weibull random variable.