**Theorem** Random variates from the discrete Weibull($p, \beta$) distribution can be generated in closed-form by inversion.

**Proof** The discrete Weibull($p, \beta$) distribution has probability mass function
\[
f(x) = (1 - p)^x - (1 - p)^{(x+1)^\beta} \quad x = 0, 1, 2, \ldots,
\]
for $0 < p < 1$ and $\beta > 0$. The cumulative distribution function is
\[
F(x) = 1 - (1 - p)^{(x+1)^\beta} \quad x = 0, 1, 2, \ldots.
\]
Equating the cumulative distribution function to $u$, where $0 < u < 1$, yields an inverse cumulative distribution function
\[
F^{-1}(u) = \left[\frac{\ln(1 - u)}{\ln(1 - p)}\right]^{1/\beta} - 1 \quad 0 < u < 1.
\]
So a closed-form variate generation algorithm using inversion for the discrete Weibull($p, \beta$) distribution is

\[
\begin{align*}
generate \ U \sim U(0, 1) \\
X &\leftarrow \left[\frac{\ln(1 - u)}{\ln(1 - p)}\right]^{1/\beta} - 1 \\
return(X)
\end{align*}
\]

**APPL failure:** The APPL statements
\[
X := [[1 - (1 - p) ^ ((x + 1) ^ beta)], [0, infinity], ["Discrete", "CDF"]]; \\
IDF(X);
\]
fail to produce the inverse distribution function of a discrete Weibull random variable.