Theorem The geometric(p) distribution is a special case of the discrete Weibull (p, β) distribution when $\beta = 1$.

Proof The discrete Weibull(p, β) distribution has probability mass function

$$f(x) = (1-p)^{x^\beta} - (1-p)^{(x+1)^\beta} \qquad x = 0, 1, 2, \ldots.$$

When $\beta = 1$, this reduces to

$$\begin{aligned} f(x) &= (1-p)^x - (1-p)^{x+1} \\ &= (1-p)^x(1-(1-p)) \\ &= p(1-p)^x \qquad x = 0, 1, 2, \ldots, \end{aligned}$$

which is the probability mass function of the geometric(p) distribution.

APPL verification: The APPL statements

```
assume(beta > 0);
assume(p > 0, p < 1);
X := [[x -> (1 - p) ^ (x ^ beta) - (1 - p) ^ ((x + 1) ^ beta)],
      [0 .. infinity], ["Discrete", "PDF"]];
subs(beta = 1, X[1][1](x));
```

verify that the geometric distribution is a special case of the discrete Weibull distribution when $\beta = 1$.