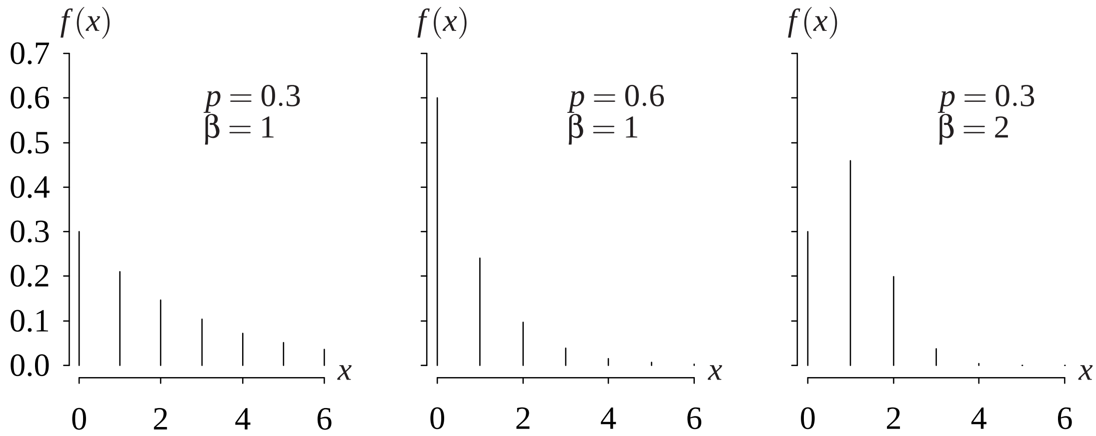
Discrete Weibull distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{Discrete Weibull}(p, \beta)$ is used to indicate that the random variable X has the discrete Weibull distribution with real parameter p satisfying $0 < p < 1$, and positive shape parameter β. A discrete Weibull random variable X with parameters p and β has probability mass function

$$f(x) = (1-p)^{x^\beta} - (1-p)^{(x+1)^\beta} \qquad x = 0, 1, 2, \ldots.$$

The probability mass function for three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - (1-p)^{(x+1)^\beta} \qquad x = 0, 1, 2, \ldots.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = (1-p)^{x^\beta} \qquad x = 0, 1, 2, \ldots.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = 1 - (1-p)^{(x+1)^\beta - x^\beta} \qquad x = 0, 1, 2, \ldots.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -x^\beta \ln(1-p) \qquad x = 0, 1, 2, \ldots.$$

The inverse distribution function of X is mathematically intractable.

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \sum_{x=0}^{\infty} \left((1-p)^{x^\beta} - (1-p)^{(x+1)^\beta}\right) e^{tx}.$$

The population mean of X is

$$E[X] = \sum_{x=0}^{\infty} \left((1-p)^{x^\beta} - (1-p)^{(x+1)^\beta} \right) x.$$

The variance, skewness, and kurtosis of X are mathematically intractable in the generalized case.

APPL verification: The APPL statements

```
X := [[x->(1-p)^(x^b)-(1-p)^((x+1)^b)], [0 .. infinity], ["Discrete", "PDF"]];
CDF(X);
SF(X);
CHF(X);
MGF(X);
Mean(X);
```

verify the cumulative distribution function, survivor function, cumulative hazard function, moment generating function, and population mean.