**Theorem** Random variates from the discrete uniform($a, b$) distribution can be generated in closed-form by inversion.

**Proof** The discrete uniform($a, b$) distribution has probability mass function

$$f(x) = \frac{1}{b - a + 1} \quad x = a, a + 1, a + 2, \ldots, b$$

for some integers $a < b$. The cumulative distribution function is

$$F(x) = \frac{x - a + 1}{b - a + 1} \quad x = a, a + 1, a + 2, \ldots, b.$$  

Equating the cumulative distribution function to $u$, where $0 < u < 1$, yields an inverse cumulative distribution function

$$F^{-1}(u) = \lfloor a + (b - a + 1)u \rfloor \quad 0 < u < 1.$$  

So a closed-form variate generation algorithm using inversion for the rectangular($n$) distribution is

```
generate $U \sim U(0, 1)$  
X ← $\lfloor a + (b - a + 1)u \rfloor$  
return(X)  
```