Theorem Random variates from the discrete uniform(a, b) distribution can be generated in closed-form by inversion.

Proof The discrete uniform(a, b) distribution has probability mass function

$$f(x) = \frac{1}{b-a+1} \qquad\qquad x = a, a+1, a+2, \ldots, b$$

for some integers $a < b$. The cumulative distribution function is

$$F(x) = \frac{x-a+1}{b-a+1} \qquad\qquad x = a, a+1, a+2, \ldots, b.$$

Equating the cumulative distribution function to u, where $0 < u < 1$, yields an inverse cumulative distribution function

$$F^{-1}(u) = \lfloor a + (b-a+1)u \rfloor \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the rectangular(n) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \lfloor a + (b-a+1)u \rfloor$
return(X)