Theorem The discrete uniform(a, b) distribution has the residual property. That is, the distribution left-truncated at some real constant c, where $a < c < b$, is also in the discrete uniform family.

Proof The discrete uniform(a, b) distribution has probability mass function

$$f(x) = \frac{1}{b - a + 1} \qquad\qquad x = a, a + 1, \ldots, b$$

and associated survivor function

$$S(x) = \frac{b - x + 1}{b - a + 1} \qquad\qquad x = a, a + 1, \ldots, b.$$

A discrete uniform(a, b) random variable that is truncated on the left at some real constant c, $a < c < b$, has survivor function

$$S_{X|X>c}(x) = \frac{S(x)}{S(c)} = \frac{\dfrac{b - x + 1}{b - a + 1}}{\dfrac{b - c + 1}{b - a + 1}} = \frac{b - x + 1}{b - c + 1} \qquad\qquad c < x < b.$$

The associated probability mass function is

$$f_{X|X>c}(x) = \frac{1}{b - c + 1} \qquad\qquad c < x < b,$$

which is in the discrete uniform family.

APPL failure: The APPL statements

```
assume(a > 0);
assume(b > 0);
additionally(a < b);
additionally(a, posint);
additionally(b, posint);
X := [[(b - x + 1) / (b - a + 1)], [a, b], ["Discrete", "SF"]];
SF(X);
assume(c > a);
additionally(c < b);
additionally(c, posint);
SF(X)[1][1](x) / SF(X)[1][1](c);
```

fail to produce the survivor function of a discrete uniform random variable.