Theorem The discrete uniform\((a,b)\) distribution has the residual property. That is, the distribution left-truncated at some real constant \(c\), where \(a < c < b\), is also in the discrete uniform family.

Proof The discrete uniform\((a,b)\) distribution has probability mass function

\[
f(x) = \frac{1}{b - a + 1} \quad x = a, a + 1, \ldots, b
\]

and associated survivor function

\[
S(x) = \frac{b - x + 1}{b - a + 1} \quad x = a, a + 1, \ldots, b.
\]

A discrete uniform\((a,b)\) random variable that is truncated on the left at some real constant \(c\), \(a < c < b\), has survivor function

\[
S_{X|X>c}(x) = \frac{S(x)}{S(c)} = \frac{\frac{b - x + 1}{b - a + 1}}{\frac{b - c + 1}{b - a + 1}} = \frac{b - x + 1}{b - c + 1} \quad c < x < b.
\]

The associated probability mass function is

\[
f_{X|X>c}(x) = \frac{1}{b - c + 1} \quad c < x < b,
\]

which is in the discrete uniform family.

**APPL failure:** The APPL statements

```
assume(a > 0);
assume(b > 0);
additionally(a < b);
additionally(a, posint);
additionally(b, posint);
X := [[(b - x + 1) / (b - a + 1)], [a, b], ["Discrete", "SF"]];
SF(X);
assume(c > a);
additionally(c < b);
additionally(c, posint);
SF(X)[1][1](x) / SF(X)[1][1](c);
```

fail to produce the survivor function of a discrete uniform random variable.