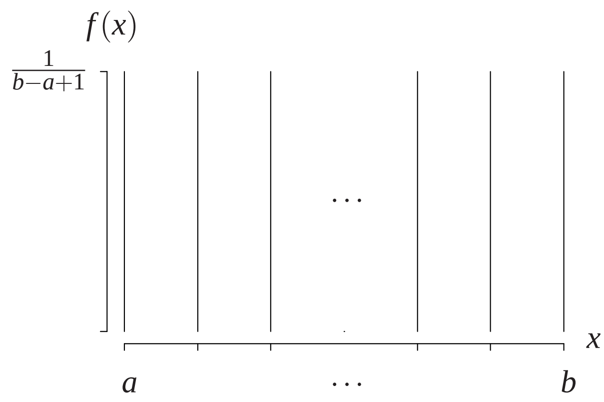
Discrete uniform distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ discrete uniform(a,b) is used to indicate that the random variable X has the discrete uniform distribution with integer parameters a and b, where $a < b$. A discrete uniform random variable X with parameters a and b has probability mass function

$$f(x) = \frac{1}{b-a+1} \qquad x = a, a+1, \ldots, b.$$

The probability mass function is illustrated below.



The cumulative distribution function is

$$F(x) = P(X \le x) = \frac{x-a+1}{b-a+1} \qquad x = a, a+1, \ldots, b.$$

The survivor function of X is

$$S(x) = P(X \ge x) = \frac{b-x+1}{b-a+1} \qquad x = a, a+1, \ldots, b.$$

The hazard function of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{1}{b-x+1} \qquad x = a, a+1, \ldots, b.$$

The inverse distribution function of X is

$$F^{-1}(u) = a + \lfloor u(b-a+1) \rfloor \qquad 0 < u < 1.$$

The median, m, of X is

$$m = \frac{a+b}{2}.$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \frac{e^{at} - e^{(b+1)t}}{(b-a+1)(1-e^t)} \qquad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \frac{e^{ait} - e^{(b+1)it}}{(b-a+1)(1-e^{it})} \qquad -\infty < t < \infty.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{a+b}{2} \qquad V[X] = \frac{(b-a+1)^2 - 1}{12}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{6\left((b-a+1)^2+1\right)}{5\left((b-a+1)^2-1\right)}.$$

APPL verification: The APPL statements

```
assume(b > a);
X:=[[x -> 1 / (b - a + 1)], [a .. b], ["Discrete", "PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
MGF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution, survivor function, hazard function, cumulative hazard function, inverse distribution function, moment generating function, population mean, variance, skewness, kurtosis.