**Discrete uniform distribution** (from [http://www.math.wm.edu/~leemis/chart/UDR/UDR.html](http://www.math.wm.edu/~leemis/chart/UDR/UDR.html))

The shorthand $X \sim \text{discrete uniform}(a, b)$ is used to indicate that the random variable $X$ has the discrete uniform distribution with integer parameters $a$ and $b$, where $a < b$. A discrete uniform random variable $X$ with parameters $a$ and $b$ has probability mass function

$$f(x) = \frac{1}{b-a+1} \quad x = a, a+1, \ldots, b.$$ 

The probability mass function is illustrated below.

The cumulative distribution function is

$$F(x) = P(X \leq x) = \frac{x-a+1}{b-a+1} \quad x = a, a+1, \ldots, b.$$ 

The survivor function of $X$ is

$$S(x) = P(X \geq x) = \frac{b-x+1}{b-a+1} \quad x = a, a+1, \ldots, b.$$ 

The hazard function of $X$ is

$$h(x) = \frac{f(x)}{S(x)} = \frac{1}{b-x+1} \quad x = a, a+1, \ldots, b.$$ 

The inverse distribution function of $X$ is

$$F^{-1}(u) = a + \lfloor u(b-a+1) \rfloor \quad 0 < u < 1.$$ 

The median, $m$, of $X$ is

$$m = \frac{a+b}{2}.$$
The moment generating function of $X$ is

$$M(t) = E[e^{tX}] = \frac{e^{at} - e^{(b+1)t}}{(b-a+1)(1-e^t)} \quad -\infty < t < \infty.$$ 

The characteristic function of $X$ is

$$\phi(t) = E[e^{itX}] = \frac{e^{ait} - e^{(b+1)it}}{(b-a+1)(1-e^{it})} \quad -\infty < t < \infty.$$ 

The population mean, variance, skewness, and kurtosis of $X$ are

$$E[X] = \frac{a+b}{2} \quad V[X] = \frac{(b-a+1)^2 - 1}{12}$$

$$E \left[ \left( \frac{X-\mu}{\sigma} \right)^3 \right] = 0 \quad E \left[ \left( \frac{X-\mu}{\sigma} \right)^4 \right] = \frac{6((b-a+1)^2 + 1)}{5((b-a+1)^2 - 1)}.$$ 

**APPL verification:** The APPL statements

```appl
assume(b > a);
X:=[[x -> 1 / (b - a + 1)], [a .. b], ["Discrete", "PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
MGF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution, survivor function, hazard function, cumulative hazard function, inverse distribution function, moment generating function, population mean, variance, skewness, kurtosis.