**Theorem** An exponential random variable with parameter $\alpha = 2$ and a chi-square random variable with $n = 2$ degrees of freedom have the same probability distribution.

**Proof** Let the random variable $X$ have the chi-square distribution with $n$ degrees of freedom with probability density function

$$f_X(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2} \qquad x > 0$$

and the random variable $Y$ have the exponential distribution with mean $\alpha$ with probability density function

$$f_Y(y) = (1/\alpha) e^{-y/\alpha} \qquad y > 0.$$

For $n = 2$

$$f_X(x) = \frac{1}{2^{2/2}\Gamma(2/2)} x^{2/2-1} e^{-x/2} = \frac{1}{2} e^{-x/2} \qquad x > 0.$$

For $\alpha = 2$

$$f_Y(y) = \frac{1}{2} e^{-y/2} \qquad y > 0.$$

Thus, $f_X(x) = f_Y(y)$ for $n = 2$ and $\alpha = 2$.

**APPL demonstration:** The APPL statements

```
ExponentialRV(1 / 2);
ChiSquareRV(2);
```

return identical probability density functions.