

Theorem The Erlang distribution is a special case of the chi-square distribution when n is even and $\alpha = 2$.

Proof Let X be a chi-square random variable with n degrees of freedom. The probability density function of X is

$$f_X(x) = \frac{x^{n/2-1}e^{-x/2}}{2^{n/2}\Gamma(n/2)} \quad x > 0.$$

Let n be even. Then $m = n/2$ is an integer. The probability density function of X becomes

$$f(x) = \frac{x^{m-1}e^{-x/2}}{2^m(m-1)!} \quad x > 0,$$

which is the probability density function of an Erlang(2, m) random variable.

APPL verification: The APPL statements

```
X := ChiSquareRV(n);  
Y := ErlangRV(2, m);  
PDF(X);  
PDF(Y);
```

confirm the result.