Theorem The square root of a chi-square(n) random variable is a chi(n) random variable.

Proof Let the random variable X have the chi-square distribution with n degrees of freedom with probability density function

$$f_X(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2} \qquad x > 0.$$

The transformation $Y = g(X) = \sqrt{X}$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = Y^2$ and Jacobian

$$\frac{dX}{dY} = 2Y.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{1}{2^{n/2}\Gamma(n/2)} (y^2)^{n/2-1} e^{-y^2/2} |2y| \\ &= \frac{1}{2^{n/2-1}\Gamma(n/2)} y^{n-1} e^{-y^2/2} \qquad y > 0, \end{aligned}$$

which is the probability density function of the chi distribution with n degrees of freedom.

APPL verification: The APPL statements

```
X := ChiSquareRV(n);
g := [[x -> sqrt(x)], [0, infinity]];
Y := Transform(X, g);
Z := ChiRV(m);
```

yield the identical functional form

$$f_Y(y) = \frac{1}{2^{n/2-1}\Gamma(n/2)} y^{n-1} e^{-y^2/2} \qquad y > 0$$

for the random variables Y and Z, which verifies that the square root of a chi-square random variable is chi random variable.