**Theorem** Let $X_i \sim \chi^2(n_i)$, for $i = 1, 2, \ldots, m$. If $X_1, X_2, \ldots, X_n$ are mutually independent random variables, then $X_1 + X_2 + \cdots + X_m \sim \chi^2(n_1 + n_2 + \cdots + n_m)$.

**Proof** The random variable $X_i$ has the chi-square distribution with $n_i$ degrees of freedom with probability density function

$$f_X(x) = \frac{1}{2^{n_i/2}\Gamma(n_i/2)} x^{n_i/2-1} e^{-x/2} \qquad x > 0,$$

for $i = 1, 2, \ldots, m$. Let the random variable $Y = \sum_{i=1}^{m} X_i$. The moment generating function for $X_i$ is

$$M_{X_i}(t) = (1-2t)^{-n_i/2} \qquad t < \frac{1}{2},$$

$i = 1, 2, \ldots, m$. The moment generating function of $Y$ is

$$\begin{aligned} E\left[e^{tY}\right] &= E\left[e^{t\left(\sum_{i=1}^{m} X_i\right)}\right] \\ &= E\left[e^{tX_1} e^{tX_2} \ldots e^{tX_m}\right] \\ &= E\left[e^{tX_1}\right] E\left[e^{tX_2}\right] \ldots E\left[e^{tX_m}\right] \\ &= (1-2t)^{-n_1/2} (1-2t)^{-n_2/2} \ldots (1-2t)^{-n_m/2} \\ &= (1-2t)^{\left(-\sum_{i=1}^{m} n_i/2\right)} \qquad t < \frac{1}{2}, \end{aligned}$$

which is the moment generating function of a chi-square random variable with $\sum_{i=1}^{n} n_i$ degrees of freedom.

**APPL illustration:** The APPL statements

```
X1 := ChiSquareRV(n1);
X2 := ChiSquareRV(n2);
Y := Convolution(X1, X2);
MGF(Y);
```

yield the moment generating function of a chi-square random variable with parameter $n_1+n_2$. By induction, it is easy to see this result generalizes to the sum of $m$ mutually independent chi-square random variables.