Theorem The Cauchy distribution has the variate generation property.

Proof The Cauchy distribution has probability density function

$$f(x) = \frac{1}{\alpha\pi\left(1+\left(\frac{x-a}{\alpha}\right)^2\right)} \qquad -\infty < x < \infty,$$

so the cumulative distribution function of the Cauchy distribution is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\alpha\pi\left(1+\left(\frac{w-a}{\alpha}\right)^2\right)} dw = \frac{1}{\pi}\arctan\left(\frac{x-a}{\alpha}\right) + \frac{1}{2} \qquad -\infty < x < \infty.$$

Equating the cumulative distribution function to u, where $0 < u < 1$, yields the inverse cumulative distribution function

$$F^{-1}(u) = \alpha\tan\left(\pi\left(u-\frac{1}{2}\right)\right) + a \qquad 0 < u < 1.$$

Simplifying using trigonometric identities,

$$F^{-1}(u) = a - \alpha\cot(\pi u) \qquad 0 < u < 1.$$

Thus a variate generation algorithm is:

generate $U \sim U(0, 1)$
$X \leftarrow a - \alpha\cot(\pi U)$
return(X)

APPL verification: The APPL statement

```
IDF(CauchyRV(a, alpha));
```

returns the appropriate inverse distribution function.