Theorem The Cauchy distribution has the scaling property. That is, if X is a Cauchy(a, α) random variable, then λX is also a Cauchy random variable with parameters λa and $\lambda\alpha$.

Proof The cumulative distribution function of a Cauchy random variable X is given by

$$\begin{aligned} F(x) &= P(X \le x) \\ &= \int_0^x \frac{1}{\alpha\pi\left[1+\left(\frac{w-a}{\alpha}\right)^2\right]}\,dw \\ &= \frac{1}{\pi}\arctan\left(\frac{x-a}{\alpha}\right)+\frac{1}{2} \qquad -\infty < x < \infty. \end{aligned}$$

So the cumulative distribution function of λX is

$$\begin{aligned} F_{\lambda X}(x) &= P(\lambda X \le x) \\ &= P(X \le x/\lambda) \\ &= \frac{1}{\pi}\arctan\left(\frac{x/\lambda-a}{\alpha}\right)+\frac{1}{2} \\ &= \frac{1}{\pi}\arctan\left(\frac{x-\lambda a}{\lambda\alpha}\right)+\frac{1}{2} \qquad -\infty < x < \infty, \end{aligned}$$

which is a Cauchy$(\lambda a, \lambda\alpha)$ random variable. Therefore, the Cauchy distribution has the scaling property.

Maple Verification: The APPL statements

```
X := CauchyRV(a, alpha);
assume(lambda > 0);
g := [[x -> lambda * x], [-infinity, infinity]];
Y := Transform(X, g);
CDF(X);
CDF(Y);
```

confirm the cumulative distribution functions given above.