Theorem [UNDER CONSTRUCTION!] The Cauchy distribution has the inverse property. That is, if $X \sim \text{Cauchy}(a, \alpha)$ then $Y = 1/X$ also has the Cauchy distribution.

Proof [UNDER CONSTRUCTION!] Let the random variable X have the Cauchy(a, α) distribution with probability density function

$$f(x) = \frac{1}{\alpha\pi[1 + ((x - a)/\alpha)^2]} \qquad -\infty < x < \infty.$$

With the exception of $X = 0$, the transformation $Y = g(X) = 1/X$ is a 1–1 transformation from $\mathcal{X} = \{x \mid -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \mid -\infty < y < \infty\}$ with inverse $X = g^{-1}(Y) = 1/Y$ and Jacobian

$$\frac{dX}{dY} = -\frac{1}{Y^2}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{1}{\alpha\pi[1 + (((1/y) - a)/\alpha)^2]} \left| \frac{1}{y^2} \right| \\ &= \frac{1}{\pi\left(\alpha y^2 + \frac{1}{\alpha y^2} - \frac{2a}{\alpha y} + \frac{a^2}{\alpha}\right)}, \end{aligned}$$

which should be the probability density function of a Cauchy random variable. The general result

$$1/X \sim \text{Cauchy}\left(\frac{a}{a^2 + \alpha^2}, \frac{\alpha}{a^2 + \alpha^2}\right)$$

appears in Forbes, Evans, Hastings, and Peacock (Statistical Distributions, fourth edition, John Wiley and Sons, 2011, page 67).