**Theorem [UNDER CONSTRUCTION!]** The Cauchy distribution has the inverse property. That is, if $X \sim \text{Cauchy}(a, \alpha)$ then $Y = 1/X$ also has the Cauchy distribution.

**Proof [UNDER CONSTRUCTION!]** Let the random variable $X$ have the Cauchy($a, \alpha$) distribution with probability density function

$$f(x) = \frac{1}{\alpha \pi [1 + ((x-a)/\alpha)^2]} \quad -\infty < x < \infty.$$  

With the exception of $X = 0$, the transformation $Y = g(X) = 1/X$ is a 1–1 transformation from $\mathcal{X} = \{x \mid -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \mid -\infty < y < \infty\}$ with inverse $X = g^{-1}(Y) = 1/Y$ and Jacobian

$$\frac{dX}{dY} = -\frac{1}{Y^2}.$$  

Therefore, by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = \frac{1}{\alpha \pi [1 + (((1/y)-a)/\alpha)^2]} \left| \frac{1}{y^2} \right| = \frac{1}{\pi \left(\alpha y^2 + \frac{1}{\alpha y^2} - \frac{2a}{\alpha y} + \frac{a^2}{\alpha} \right)},$$

which should be the probability density function of a Cauchy random variable. The general result

$$1/X \sim \text{Cauchy} \left( \frac{a}{a^2 + \alpha^2}, -\frac{\alpha}{a^2 + \alpha^2} \right)$$