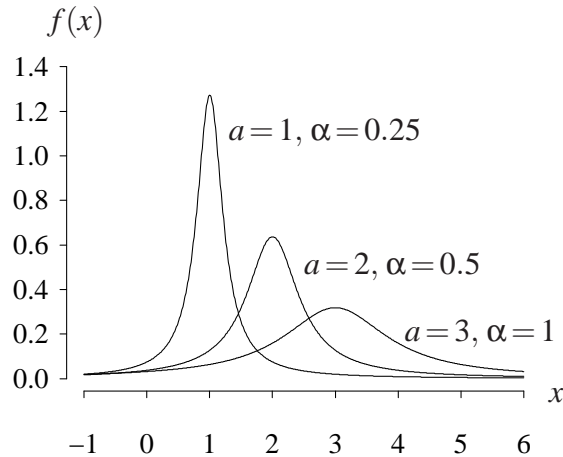


Cauchy distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{Cauchy}(a, \alpha)$ is used to indicate that the random variable X has the Cauchy distribution with positive scale parameter α and location parameter a . A Cauchy random variable X with parameters a and α has probability density function

$$f(x) = \frac{1}{\alpha\pi[1 + ((x-a)/\alpha)^2]} \quad -\infty < x < \infty$$

for $a > 0$ and $-\infty < a < \infty$. The Cauchy distribution is of interest because its moments are undefined. The probability density function for various combinations of a and α is given below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = \frac{1}{2\pi} \left(\pi - 2 \arctan \left(\frac{a-x}{\alpha} \right) \right) \quad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = \frac{1}{2\pi} \left(\pi + 2 \arctan \left(\frac{a-x}{\alpha} \right) \right) \quad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = 2\alpha (\alpha^2 - 2xa + a^2 + x^2)^{-1} \left(\pi + 2 \arctan \left(\frac{a-x}{\alpha} \right) \right)^{-1} \quad -\infty < x < \infty.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \ln(2\pi) - \ln \left(\pi + 2 \arctan \left(\frac{a-x}{\alpha} \right) \right) \quad -\infty < x < \infty.$$

The inverse distribution function of X is

$$F^{-1}(u) = a - \alpha \cot(\pi u) \quad 0 < u < 1.$$

The population median of X is the location parameter, a .

The population moments of X are undefined. It follows that the population mean, variance, skewness, and kurtosis of X are also undefined.

APPL verification: The APPL statements

```
X := CauchyRV(a, alpha);  
CDF(X);  
SF(X);  
HF(X);  
IDF(X);  
Mean(X);  
Variance(X);  
Skewness(X);  
Kurtosis(X);  
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, mean, variance, skewness, kurtosis, and moment generating function.