Theorem The Poisson(μ) distribution is the limit of the binomial(n, p) distribution with $\mu = np$ as $n \to \infty$.

Proof Let the random variable X have the binomial(n, p) distribution. Replacing p with μ/n (which will be between 0 and 1 for large n),

$$\begin{aligned} f(x) &= \binom{n}{x} p^x (1-p)^{n-x} \\ &= \frac{n(n-1)(n-2)\ldots(n-x+1)}{x!} \left(\frac{\mu}{n}\right)^x \left(1-\frac{\mu}{n}\right)^{n-x} \\ &= \frac{n}{n} \cdot \frac{n-1}{n} \cdot \ldots \cdot \frac{n-x+1}{n} \cdot \frac{\mu^x}{x!} \left(1-\frac{\mu}{n}\right)^n \left(1-\frac{\mu}{n}\right)^{-x} \qquad x = 0, 1, \ldots, n. \end{aligned}$$

Taking the limit as $n \to \infty$ yields

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \qquad x = 0, 1, 2 \ldots,$$

which is the probability mass function for the Poisson(μ) distribution.

APPL failure: Since `BinomialRV` did not accept μ/n as a second, argument, the APPL list-of-sublists was keyed in directly as:

```
assume(n, posint);
assume(mu > 0);
X := [[x -> n! / (x! * (n - x)!) * (mu / n) ^ x * (1 - mu / n)
           ^ (n - x)], [0 .. n], ["Discrete", "PDF"]];
limit(X[1][1](x), n = infinity);
```

Maple was not able to evaluate the limit. On the other hand, manually entering the following Maple statement confirms the derivation

```
assume(x, integer);
additionally(x >= 0);
assume(mu > 0);
limit(n! / (x! * (n - x)!) * (mu / n) ^ x * (1 - mu / n) ^ (n - x), n = infinity);
```