Theorem For $X \sim \text{binomial}(n, p)$, as $n \to \infty$, $X \overset{a}{\sim} N(np, np(1-p))$.

Proof Let $X_i \sim \text{Bernoulli}(p)$, for $i = 1, 2, \ldots, n$. Then $X = \sum_{i=1}^{n} X_i \sim \text{binomial}(n, p)$, for $X_1, X_2, \ldots, X_n$ mutually independent random variables.

As $n \to \infty$, by the Central Limit Theorem,

$$\lim_{n\to\infty} \frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \sim N(0, 1)$$

or

$$\lim_{n\to\infty} \frac{X_1 + X_2 + \cdots + X_n - np}{\sqrt{np(1-p)}} \sim N(0, 1).$$

Hence

$$X \overset{a}{\sim} N(np, np(1-p))$$

as a special case of the central limit theorem.