Theorem If $X_i \sim \text{binomial}(n_i, p)$ for $i = 1, 2, \ldots, m$, then $\sum_{i=1}^{n} X_i \sim \text{binomial}\left(\sum_{i=1}^{m} n_i, p\right)$.

Proof The probability mass function of X_i is

$$f_{X_i}(x_i) = \binom{n_i}{x_i} p^{x_i}(1-p)^{1-x_i} \qquad x = 0, 1, 2, \ldots, n_i,$$

for $i = 1, 2, \ldots, m$ and $-\infty < t < \infty$. The moment generating function for a binomial random variable is

$$M_{X_i}(t) = \left(pe^t + (1-p)\right)^{n_i}$$

for $i = 1, 2, \ldots, m$. Let the random variable $Y = \sum_{i=1}^{m} X_i$. The moment generating function of Y is

$$\begin{aligned}
E\left[e^{tY}\right] &= E\left[e^{t\left(\sum_{i=1}^{m} X_i\right)}\right] \\
&= E\left[e^{tX_1} e^{tX_2} \ldots e^{tX_n}\right] \\
&= E\left[e^{tX_1}\right] E\left[e^{tX_2}\right] \ldots E\left[e^{tX_n}\right] \\
&= \left(pe^t + (1-p)\right)^{n_1} \left(pe^t + (1-p)\right)^{n_2} \ldots \left(pe^t + (1-p)\right)^{n_m} \\
&= \left(pe^t + (1-p)\right)^{\sum_{i=1}^{m} n_i}
\end{aligned}$$

for $-\infty < t < \infty$, which is the moment generating function of a binomial random variable with parameters $\sum_{i=1}^{m} n_i$ and p.

APPL illustration: The APPL statements

```
X1 := BinomialRV(n1, p);
X2 := BinomialRV(n2, p);
Y := Convolution(X1, X2);
MGF(Y);
```

fail to provide the appropriate moment generating function.