Theorem The rectangular(n) distribution is a special case of the beta-binomial(a, b, n) distribution with $a = b = 1$.

Proof The beta-binomial distribution has probability density function

$$f(x) = \frac{\Gamma(x+a)\Gamma(n-x+b)\Gamma(a+b)\Gamma(n+2)}{(n+1)\Gamma(a+b+n)\Gamma(a)\Gamma(b)\Gamma(x+1)\Gamma(n-x+1)} \qquad x = 0, 1, 2, \ldots, n.$$

Substituting $a = b = 1$ yields

$$\begin{aligned} f(x) &= \frac{\Gamma(x+1)\Gamma(n-x+1)\Gamma(2)\Gamma(n+2)}{(n+1)\Gamma(2+n)\Gamma(1)\Gamma(1)\Gamma(x+1)\Gamma(n-x+1)} \\ &= \frac{\Gamma(2)}{n+1} \\ &= \frac{1}{n+1} \qquad x = 0, 1, 2, \ldots, n, \end{aligned}$$

which is the probability density function of the rectangular distribution.

APPL verification: The APPL statements

```
X := [[x -> (GAMMA(x + a) * GAMMA(n - x + b) * GAMMA(a + b) * GAMMA(n + 2)) /
            ((n + 1) * GAMMA(a + b + n) * GAMMA(a) * GAMMA(b) * GAMMA(x + 1) *
            GAMMA(n - x + 1))], [0, n], ["Discrete", "PDF"]];
simplify(subs([a = 1, b = 1], %));
```

yield a probability density function that is identical to that of the rectangular distribution. This verifies that the rectangular distribution is a special case of the beta-binomial distribution when $a = 1$ and $b = 1$.