Theorem The negative hypergeometric distribution is a special case of the beta-binomial distribution when \( a = n_1, \ b = n_3 - n_1, \) and \( n = n_2. \) [This result is incorrectly stated on the chart. The error was detected and corrected by Jean Peyhardi at the University of Montpellier in October of 2017. Thank you Professor Peyhardi!]

Proof Let the random variable \( X \sim \text{beta-binomial}(a, b, n) \). The probability mass function of \( X \) is

\[
f(x) = \frac{\Gamma(x + a) \Gamma(n - x + b) \Gamma(a + b) \Gamma(n + 2)}{(n + 1) \Gamma(a + b + n) \Gamma(a) \Gamma(b) \Gamma(x + 1) \Gamma(n - x + 1)} \quad x = 0, 1, \ldots, n.
\]

Substituting \( a = n_1, \ b = n_3 - n_1, \) and \( n = n_2 \) yields

\[
f(x) = \frac{\Gamma(x + n_1) \Gamma(n_2 - x + n_3 - n_1) \Gamma(n_3) \Gamma(n_2 + 2)}{(n_2 + 1) \Gamma(n_2 + n_3) \Gamma(n_1) \Gamma(n_3 - n_1) \Gamma(x + 1) \Gamma(n_2 - x + 1)} \quad x = 0, 1, \ldots, n_2.
\]

This reduces to

\[
f(x) = \frac{(x + n_1 - 1)! (n_2 - x + n_3 - n_1 - 1)! (n_3 - 1)! (n_2)!}{(n_2 + n_3 - 1)! (n_1 - 1)! (n_3 - n_1 - 1)! (x)! (n_2 - x)!} \quad x = 0, 1, \ldots, n_2
\]

because \( \Gamma(n) = (n - 1)! \) when \( n \) is an integer. This reduces to

\[
f(x) = \frac{(n_1 + x - 1)}{x} \frac{(n_3 - n_1 + n_2 - x - 1)}{(n_2 - x) \binom{n_3 + n_2 - 1}{n_2}} \quad x = 0, 1, \ldots, n_2,
\]

which is the probability mass function of a negative hypergeometric\((n_1, n_2, n_3)\) random variable.