Theorem The negative hypergeometric distribution is a special case of the beta-binomial distribution when $a = n_1$, $b = n_3 - n_1$, and $n = n_2$. [This result is incorrectly stated on the chart. The error was detected and corrected by Jean Peyhardi at the University of Montpellier in October of 2017. Thank you Professor Peyhardi!]

Proof Let the random variable $X \sim$ beta-binomial(a, b, n). The probability mass function of X is

$$f(x) = \frac{\Gamma(x+a)\,\Gamma(n-x+b)\,\Gamma(a+b)\,\Gamma(n+2)}{(n+1)\,\Gamma(a+b+n)\,\Gamma(a)\,\Gamma(b)\,\Gamma(x+1)\,\Gamma(n-x+1)} \qquad x = 0, 1, \ldots, n.$$

Substituting $a = n_1$, $b = n_3 - n_1$, and $n = n_2$ yields

$$f(x) = \frac{\Gamma(x+n_1)\,\Gamma(n_2-x+n_3-n_1)\,\Gamma(n_3)\,\Gamma(n_2+2)}{(n_2+1)\,\Gamma(n_2+n_3)\,\Gamma(n_1)\,\Gamma(n_3-n_1)\,\Gamma(x+1)\,\Gamma(n_2-x+1)} \qquad x = 0, 1, \ldots, n_2.$$

This reduces to

$$f(x) = \frac{(x+n_1-1)!\,(n_2-x+n_3-n_1-1)!\,(n_3-1)!\,(n_2)!}{(n_2+n_3-1)!\,(n_1-1)!\,(n_3-n_1-1)!\,(x)!\,(n_2-x)!} \qquad x = 0, 1, \ldots, n_2$$

because $\Gamma(n) = (n-1)!$ when n is an integer. This reduces to

$$f(x) = \frac{\binom{n_1+x-1}{x}\binom{n_3-n_1+n_2-x-1}{n_2-x}}{\binom{n_3+n_2-1}{n_2}} \qquad x = 0, 1, \ldots, n_2,$$

which is the probability mass function of a negative hypergeometric(n_1, n_2, n_3) random variable.