Theorem The distribution of a beta random variable with parameters $\beta = 1/2$, $\gamma = 1/2$ follows the arcsin distribution.

Proof Let X be a beta random variable with parameters β and γ. Then by definition, X has probability density function

$$f(x) = \left[\frac{\Gamma(\beta+\gamma)}{\Gamma(\beta)\Gamma(\gamma)}\right] x^{\beta-1}(1-x)^{\gamma-1} \qquad 0 < x < 1.$$

Now set $\beta = \gamma = 1/2$, and the resulting probability density function is

$$f(x) = \left[\frac{\Gamma(1)}{\Gamma(1/2)\Gamma(1/2)}\right] x^{-1/2}(1-x)^{-1/2} \qquad 0 < x < 1.$$

Substituting $\Gamma(1) = 0! = 1$ and $\Gamma(1/2) = \sqrt{\pi}$, $f(x)$ becomes

$$f(x) = \frac{1}{\pi\sqrt{x(1-x)}} \qquad 0 < x < 1,$$

which is the probability density function of an arcsin random variable.

APPL verification: The APPL statements

```
BetaRV(1 / 2, 1 / 2);
ArcSinRV();
```

produce the same probability density function, verifying the result.