**Theorem**  The distribution of a beta random variable with parameters \( \beta = 1/2, \gamma = 1/2 \) follows the arcsin distribution.

**Proof**  Let \( X \) be a beta random variable with parameters \( \beta \) and \( \gamma \). Then by definition, \( X \) has probability density function

\[
f(x) = \left[ \frac{\Gamma(\beta + \gamma)}{\Gamma(\beta)\Gamma(\gamma)} \right] x^{\beta - 1}(1 - x)^{\gamma - 1} \quad 0 < x < 1.
\]

Now set \( \beta = \gamma = 1/2 \), and the resulting probability density function is

\[
f(x) = \left[ \frac{\Gamma(1)}{\Gamma(1/2)\Gamma(1/2)} \right] x^{-1/2}(1 - x)^{-1/2} \quad 0 < x < 1.
\]

Substituting \( \Gamma(1) = 0! = 1 \) and \( \Gamma(1/2) = \sqrt{\pi} \), \( f(x) \) becomes

\[
f(x) = \frac{1}{\pi \sqrt{x(1 - x)}} \quad 0 < x < 1,
\]

which is the probability density function of an arcsin random variable.

**APPL verification:**  The APPL statements

\[
\text{BetaRV}(1 / 2, 1 / 2);
\text{ArcSinRV};
\]

produce the same probability density function, verifying the result.