Theorem The minimum of n mutually independent Bernoulli random variables is a Bernoulli random variable.

Proof A Bernoulli(p) random variable has probability mass function

$$f(x) = P(X = x) = \begin{cases} 1 - p & x = 0 \\ p & x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let X_i be Bernoulli(p_i) for $i = 1, 2, \ldots, n$. Then,

$$Y = \min\{X_1, X_2, \ldots, X_n\} \sim \text{Bernoulli}\left(\prod_{i=1}^{n} p_i\right)$$

because the support of Y is $\{0,1\}$ and $Y = 1$ occurs only when $X_i = 1$ for $i = 1, 2, \ldots, n$. Because of the assumption of mutual independence, the p_i values are multiplied.

APPL verification: The APPL statements

```
MinimumIID(BernoulliRV(p), n);
simplify(op(%[1])[1]);
```

ultimately yield the expressions p^n for $x = 1$ and $1 - p^n$ for $x = 0$, which form a Bernoulli distribution with parameter p^n. This illustrates a special case of the result when the n random variables are identically distributed. Likewise, the APPL statements

```
X1 := BernoulliRV(p1);
X2 := BernoulliRV(p2);
Minimum(X1, X2);
```

illustrate the result for $n = 2$.