Theorem If $X_1, X_2, \ldots, X_n$ are mutually independent Bernoulli(p) random variables then $Y = \sum_{i=1}^{n} X_i$ is binomial(n, p).

Proof The moment generating function of X_i is

$$M_{X_i}(t) = 1 - p + pe^t$$

for $i = 1, 2, \ldots, n$ and $-\infty < t < \infty$. So the moment generating function of Y is

$$\begin{aligned} M_Y(t) &= \prod_{i=1}^{n} \left(1 - p + pe^t\right) \\ &= \left(1 - p + pe^t\right)^n \end{aligned}$$

for $-\infty < t < \infty$, which is the moment generating function of a binomial(n, p) random variable.

APPL illustration: The APPL statements

```
X := BernoulliRV(1 / 2);
Z := ConvolutionIID(X, 3);
BinomialRV(3 , 1 / 2);
```

produces output that is consistent with the result.