Bernoulli distribution (from [http://www.math.wm.edu/~leemis/chart/UDR/UDR.html](http://www.math.wm.edu/~leemis/chart/UDR/UDR.html))

The shorthand $X \sim \text{Bernoulli}(p)$ is used to indicate that the random variable $X$ has the Bernoulli distribution with parameter $p$, where $0 < p < 1$. A Bernoulli random variable $X$ with success probability $p$ has probability mass function

$$f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$$

for $0 < p < 1$. The Bernoulli distribution is associated with the notion of a Bernoulli trial, which is an experiment with two outcomes, generically referred to as success ($x = 1$) and failure ($x = 0$). The cumulative distribution function of $X \sim \text{Bernoulli}(p)$ is

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 0 \\ 1 - p & 0 \leq x < 1 \\ 1 & x \geq 1. \end{cases}$$

The survivor function of $X$ is

$$S(x) = P(X \geq x) = \begin{cases} 1 & x \leq 0 \\ p & 0 < x \leq 1 \\ 0 & x > 1. \end{cases}$$

The hazard function of $X$ on the support is

$$h(x) = \frac{f(x)}{S(x)} = \begin{cases} 1 - p & x = 0 \\ 1 & x = 1. \end{cases}$$

The cumulative hazard function of $X$ on $x \leq 1$ is

$$H(x) = -\ln S(x) = \begin{cases} 0 & x \leq 0 \\ -\ln p & 0 < x \leq 1. \end{cases}$$

The inverse distribution function of $X$ is

$$F^{-1}(u) = \begin{cases} 0 & 0 < u < 1 - p \\ 1 & 1 - p \leq u < 1. \end{cases}$$

The median of $X$ is 0 if $0 < p \leq 1/2$ and 1 if $1/2 < p < 1$. The mode of $X$, denoted by $m$, is

$$m = \begin{cases} 0 & 0 < p < 1/2 \\ 1 & 1/2 < p < 1. \end{cases}$$

The moment generating function of $X$ is

$$M(t) = E \left[ e^{tX} \right] = (1 - p) + pe^t, \quad -\infty < t < \infty.$$
**APPL verification:** The APPL statements

\[
X := \text{BernoulliRV}(p);
\]
\[
\text{CDF}(X);
\]
\[
\text{SF}(X);
\]
\[
\text{HF}(X);
\]
\[
\text{CHF}(X);
\]
\[
\text{IDF}(X);
\]
\[
\text{Mean}(X);
\]
\[
\text{Variance}(X);
\]
\[
\text{Skewness}(X);
\]
\[
\text{Kurtosis}(X);
\]
\[
\text{MGF}(X);
\]

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.