

Bernoulli distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{Bernoulli}(p)$ is used to indicate that the random variable X has the Bernoulli distribution with parameter p , where $0 < p < 1$. A Bernoulli random variable X with success probability p has probability mass function

$$f(x) = p^x(1-p)^{1-x} \quad x = 0, 1$$

for $0 < p < 1$. The Bernoulli distribution is associated with the notion of a *Bernoulli trial*, which is an experiment with two outcomes, generically referred to as *success* ($x = 1$) and *failure* ($x = 0$). The cumulative distribution function of $X \sim \text{Bernoulli}(p)$ is

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 0 \\ 1-p & 0 \leq x < 1 \\ 1 & x \geq 1. \end{cases}$$

The survivor function of X is

$$S(x) = P(X \geq x) = \begin{cases} 1 & x \leq 0 \\ p & 0 < x \leq 1 \\ 0 & x > 1. \end{cases}$$

The hazard function of X on the support is

$$h(x) = \frac{f(x)}{S(x)} = \begin{cases} 1-p & x = 0 \\ 1 & x = 1. \end{cases}$$

The cumulative hazard function of X on $x \leq 1$ is

$$H(x) = -\ln S(x) = \begin{cases} 0 & x \leq 0 \\ -\ln p & 0 < x \leq 1. \end{cases}$$

The inverse distribution function of X is

$$F^{-1}(u) = \begin{cases} 0 & 0 < u < 1-p \\ 1 & 1-p \leq u < 1. \end{cases}$$

The median of X is 0 if $0 < p \leq 1/2$ and 1 if $1/2 < p < 1$. The mode of X , denoted by m , is

$$m = \begin{cases} 0 & 0 < p < 1/2 \\ 1 & 1/2 < p < 1. \end{cases}$$

The moment generating function of X is

$$M(t) = E[e^{tX}] = (1-p) + pe^t \quad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = E[e^{itX}] = (1-p) + pe^{it} \quad -\infty < t < \infty.$$

The population mean, variance, skewness, and kurtosis of X are

$$\begin{aligned} E[X] &= p & V[X] &= p(1-p) \\ E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] &= \frac{1-2p}{\sqrt{p(1-p)}} & E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] &= \frac{3p^2-3p+1}{p(1-p)}. \end{aligned}$$

APPL verification: The APPL statements

```
X := BernoulliRV(p);  
CDF(X);  
SF(X);  
HF(X);  
CHF(X);  
IDF(X);  
Mean(X);  
Variance(X);  
Skewness(X);  
Kurtosis(X);  
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.