Theorem Random variates from the arctangent distribution with parameters λ and ϕ can be generated in closed-form by inversion.

Proof The arctangent(λ, ϕ) distribution has cumulative distribution function

$$F(x) = 2\left(\frac{\arctan(\lambda\,\phi) - \arctan(-x\lambda + \lambda\,\phi)}{2\,\arctan(\lambda\,\phi) + \pi}\right) \qquad x \geq 0.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \frac{\lambda\phi + \tan[-\arctan(\lambda\phi) + u\pi/2 + u\arctan(\lambda\phi)]}{\lambda} \qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the arctangent(λ, ϕ) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow (\lambda\phi + \tan[-\arctan(\lambda\phi) + U\pi/2 + U\arctan(\lambda\phi)])\,/\lambda$
return(X)

APPL verification: The APPL statements

```
X := ArcTanRV(lambda, phi);
CDF(X);
IDF(X);
```

verify the inverse distribution function of an arctangent random variable.