**Theorem** The arcsin distribution has the variate generation property, i.e., its cumulative distribution function can be inverted in closed form.

**Proof** Let $X$ be an arcsin random variable. The probability density function of $X$ is

$$f_X(x) = \frac{1}{\pi\sqrt{x(1-x)}} \qquad 0 < x < 1.$$

We first find the cumulative distribution function of $X$:

$$\begin{aligned}
F_X(x) &= \int_0^x f_X(t)\,dt \\
&= \int_0^x \frac{1}{\pi\sqrt{t(1-t)}}\,dt \\
&= \frac{1}{\pi}\int_0^x \frac{1}{\sqrt{t(1-t)}}\,dt \\
&= \frac{1}{\pi}\int_{-1}^{2x-1} \frac{1}{\sqrt{1-u^2}}\,du \qquad \text{(after making a } u = 2t-1 \text{ substitution)} \\
&= \frac{1}{\pi}\arcsin(u)\Big|_{-1}^{2x-1} \\
&= \frac{1}{\pi}\left[\arcsin(2x-1) - \arcsin(-1)\right] \\
&= \frac{1}{\pi}\left[\arcsin(2x-1) + \frac{\pi}{2}\right] \\
&= \frac{\arcsin(2x-1)}{\pi} + \frac{1}{2} \qquad 0 < x < 1.
\end{aligned}$$

We want to fix $u \in (0, 1)$ and solve for $x$:

$$\begin{aligned}
\frac{\arcsin(2x-1)}{\pi} + \frac{1}{2} &= u \\
\arcsin(2x-1) &= \pi\left(u - \frac{1}{2}\right) \\
x &= \left[\sin(\pi(u-1/2)) + 1\right]/2.
\end{aligned}$$

This verifies the variate generation property of the arcsin distribution.

**APPL verification:** The APPL statements below and a little algebra verify the result.

```
X := ArcSinRV();
IDF(X);
```