23.24 Write an R function named `firstcol` with a single argument `A`, which is a matrix consisting of numerical values. The function should return the column number of the first column containing all identical elements. If no such column exists, `firstcol` should return \(-1\).

23.25 Write a function named `GraphicArts` with a single argument `n`, a positive integer. This function should plot `n` equally-spaced points around the unit circle and connect all pairs of points with a line. A call to `GraphicArts` with an argument of 12, for example, should generate a plot like the one shown below.

![Plot of equally-spaced points around the unit circle](image)

23.26 Consider the logistic mapping

\[ f(x) = rx(1-x). \]

The logistic mapping is used by mathematical biologists to model population sizes. The iterative equation

\[ x_{n+1} = f(x_n) \]

is iterated to produce \(x_1, x_2, \ldots, x_{1000}\), for some initial condition \(0 < x_0 < 1\). The first 900 of these values are discarded, leaving \(x_{901}, x_{902}, \ldots, x_{1000}\). Make a plot of \(x_{901}, x_{902}, \ldots, x_{1000}\) (on the vertical axis) versus \(r\) (on the horizontal axis), for \(r\) values ranging from \(r = 2.6\) to \(r = 3.8\) in steps of 0.001.

23.27 Write an R function named `primeFactorization` with a positive integer argument `n`, which returns a vector whose elements are the factors in the prime factorization of `n`. Use your function to find the prime factorizations of 1234 and 12345. Based on the prime factorizations, what is the greatest common divisor of 1234 and 12345?

23.28 Let the random variable \(X\) be the length of the longest run of heads in ten tosses of a fair coin. Write R commands to find probability mass function of \(X\).

23.29 Write R commands to calculate

\[
\frac{3}{4} + \left( \frac{3}{4} \cdot \frac{5}{6} \right) + \left( \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \right) + \cdots + \left( \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdots \frac{49}{50} \right)
\]

to ten-digit accuracy.
23.30 The positive integers $a$, $b$, and $c$ are known as a Pythagorean triple if $a^2 + b^2 = c^2$. Plot the pairs $(a, b)$ associated with Pythagorean triples for values of $a$ and $b$ ranging from 1 to 1000. Make a second plot for values of $a$ and $b$ ranging from 1 to 5000. Comment on any patterns that you observe.

23.31 The R code below contains two nested for loops. Rewrite the code in a fashion so that it executes faster.

```r
> x = matrix(rnorm(50), 5, 10)
> for (i in 1:5) {
+   for (j in 1:10) {
+     x[i, j] = x[i, j] + cos(2 * pi / 3) + rnorm(1, 0, 3)
+   }
+}
```

23.32 In a file that consists of text, there are transitions between vowels and consonants in every word in the text. Write a computer program that scans in Abraham Lincoln’s Gettysburg Address and prints the number of

- vowel to vowel transitions,
- vowel to consonant transitions,
- consonant to vowel transitions, and
- consonant to consonant transitions

within words.