A digraph D is primitive if there exists an integer m such that any two not necessarily distinct vertices u and v are joined by a directed path of length m. The smallest such m is called the exponent of D and denoted by $\gamma(D)$.

A primitive digraph D has large exponent if $\gamma(D)$ satisfies $\alpha_n = \lfloor w_n/2 \rfloor + 2 \leq \gamma(D) \leq w_n$, where $w_n = (n-1)^2 + 1$. It is shown that the minimum number of arcs in a primitive digraph D on n vertices with exponent equal to α_n is either $n+1$ or $n+2$. For given $n \geq 8$, there always exists a primitive digraph on n vertices with exponent α_n and $n+2$ arcs. Such digraphs for the cases n even and n odd are different. An algorithm determines for a given n, whether the minimum number of arcs in such digraphs is $n+1$ or $n+2$.

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