A digraph $D$ is primitive if there exists an integer $m$ such that any two
not necessarily distinct vertices $u$ and $v$ are joined by a directed path of
length $m$. The smallest such $m$ is called the exponent of $D$ and denoted by
$\gamma(D)$.

A primitive digraph $D$ has large exponent if $\gamma(D)$ satisfies
$\alpha_n = \lfloor \frac{w_n}{2} \rfloor + 2 \leq \gamma(D) \leq w_n$, where $w_n = (n - 1)^2 + 1$. It is shown that the minimum
number of arcs in a primitive digraph $D$ on $n$ vertices with exponent equal
to $\alpha_n$ is either $n+1$ or $n+2$. For given $n \geq 8$, there always exists a primitive
digraph on $n$ vertices with exponent $\alpha_n$ and $n+2$ arcs. Such digraphs for the
cases $n$ even and $n$ odd are different. An algorithm determines for a given
$n$, whether the minimum number of arcs in such digraphs is $n+1$ or $n+2$.

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