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Nullity of Measurement-induced Nonlocality

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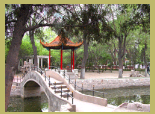

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- The outline of this talk:

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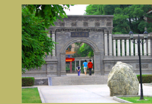
2. Measurement-induced nonlocality(MiN), quantum discord(QD) and classical-quantum(CQ) state for infinite dimension

3. Main result: Nullity of measurement-induced nonlocality



• Abstract

Measurement-induced nonlocality is a measure of nonlocality introduced by Luo and Fu [Phys. Rev. Lett. **106**, 120401(2011)]. We present a sufficient and necessary condition for nullity of measurement-induced nonlocality for both finite- and infinite-dimensional bipartite systems. We highlight the relation between zero measurement-induced nonlocality states and classical-quantum states (which have zero quantum discord) in terms of commutativity. It is indicated that measurement-induced nonlocality and quantum discord are raised from noncommutativity rather than entanglement. We find that the set of states with zero measurement-induced nonlocality is a proper subset of the set of zero discordant states, and that they are zero-measure sets. Therefore, there exist not only quantum nonlocality without entanglement but also quantum nonlocality without quantum discord.

1 Introduction

- Quantum nonlocality, whereby particles of spatially separated quantum systems can instantaneously influence one another, is one of the most elusive features in quantum theory.

- There are several kinds of nonlocalities, such as entanglement, quantum discord and measurement-induced nonlocality. They can also be viewed as quantum correlations.

- Mathematically, quantumness is always associated with *noncommutativity* while classical mechanics displays *commutativity* in some sense.

- The quantifying of nonlocality, for instance, entanglement measure and computation of quantum discord, has been discussed intensively. The aim of this work is to characterize and compare MiN, CQ and QD in terms of noncommutativity mathematically.

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- We consider a bipartite quantum system consisting of two parts labeled by A and B respectively, let H_A be the state space of the subsystem A and H_B be the state space of the subsystem B, $\dim H_A \otimes H_B \leq +\infty$.

- Mathematically, a state of the system A+B is described by a *density operator* ρ acting on the state space $H_A \otimes H_B$, namely,

$$\rho \text{ is positive, and } \mathrm{Tr}(\rho) = 1, \ \rho \in \mathcal{B}(H_A \otimes H_B).$$

- We recall some definitions for finite-dimensions.

Measurement-induced nonlocality (MiN, for short) was firstly proposed by Luo and Fu [1]. The MiN of ρ, denoted by $N(\rho)$, is defined by [1](Note:finite-dimension!)

$$N(\rho) = \max_{\Pi^A} \|\rho - \Pi^A(\rho)\|_2^2, \tag{1}$$

where $\|\cdot\|_2$ is the Hilbert-Schmidt norm (that is $\|A\|_2 = [\mathrm{Tr}(A^\dagger A)]^{\frac{1}{2}}$), and the max is taken over all local von Neumann measurement $\Pi^A = \{\Pi_k^A\}$ with $\sum_k \Pi_k^A \rho_A \Pi_k^A = \rho_A$, $\Pi^A(\rho) = \sum_k (\Pi_k^A \otimes I_B)\rho(\Pi_k^A \otimes I_B)$.

[1] S.-L. Luo and S.-S. Fu, *Measurement-induced nonlocallity*, Phys. Rev. Lett. **106**, 120401(2011).

- MiN is different from, and in some sense dual to, the geometric measure of quantum discord(GMQD) [1](Note:finite-dimension!)

$$D_G(\rho) := \min_{\Pi^A} \|\rho - \Pi^A(\rho)\|_2^2$$

where Π^A runs over *all* local von Neumann measurements (GMQD is originally introduced in [2] as $D_G(\rho) := \min_\chi \|\rho - \chi\|_2^2$ with χ runs over all zero QD states and proved in [3] that the two equations coincide).

[2] B. Dakić *et al*, *Necessary and sufficient conditiong for nonzero quatnum discord*, Phys. Rev. Lett. **105**, 190502(2010).

[3] S.-L. Luo and S.-S. Fu, *Geometric measure of quantum discord*, Phys. Rev. A **82**, 034302(2010).

- We recall that the quantum discord, which can be viewed as a measure of the minimal loss of correlation in the sense of quantum mutual information, is defined by [4](Note:finite-dimension!)

$$D(\rho) = \min_{\Pi^A}\{I(\rho) - I(\rho|\Pi^A)\}, \tag{2}$$

where the min is taken over all local von Neumann measurements Π^A. $I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho)$ is interpreted as the quantum mutual information, where $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$ is the von Neumann entropy, $I(\rho|\Pi^A)\} := S(\rho_B) - S(\rho|\Pi^A)$, $S(\rho|\Pi^A) := \sum_k p_k S(\rho_k)$, and $\rho_k = \frac{1}{p_k}(\Pi_k^A \otimes I_B)\rho(\Pi_k^A \otimes I_B)$ with $p_k = \mathrm{Tr}[(\Pi_k^A \otimes I_B)\rho(\Pi_k^A \otimes I_B)]$, $k = 1, 2, \ldots, \dim H_A$. Throughout this talk, all logarithms are taken to base 2.

[4] H. Ollivier and W.H. Zurek, *Quantum discord: a measure of the quantumness of correlations*, Phys. Rev. Letters, **88**, 017901(2001).

- QD is nonnegative [4-5].
- For finite-dimensional case, it is known that a state has zero QD if and only if it is a classical-quantum(CQ) state, where a state ρ is said to be a CQ state if it has the form of(Note:finite-dimension!)

$$\rho = \sum_i p_i |i\rangle\langle i| \otimes \rho_i^B, \tag{3}$$

for some orthonomal basis $\{|i\rangle\}$ of H_A, where ρ_i^Bs are states of the subsystem B, $p_i \geq 0$, $\sum_i p_i = 1$.

[5] A. Datta, *A condition for the nullity of quantum discord*, arXiv: 1003.5256v2(2010).

2 MiN, QD and CQ states for infinite-dimension

- The following results is based on

  Y. Guo, J.-C. Hou, *Nullity of measurement-induced non-locality*, arXiv:1107.0355v1(2011).

- With the same spirit as that of the finite-dimensional case, we can generalize MiN, QD and CQ states to infinite-dimensional case straightforward.

- In this section, we always assume that $\dim H_A \otimes H_B = +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. Let $\Pi^A = \{\Pi_k^A = |k\rangle\langle k|\}$ be a set of mutually orthogonal rank-one projections that sum up to the identity of H_A(we also call $\Pi^A = \{\Pi_k^A\}$ a local von Neumann measurement). Where $\sum_k (\Pi_k^A \otimes I_B)^\dagger (\Pi_k^A \otimes I_B) = \sum_k \Pi_k^A \otimes I_B = I_{AB}$, the series converges under the strongly operator topology [6].

  [6] J.-C. Hou, *A characterization of positive linear maps and criteria for entangled quantum states*, J. Phys A: Math. Theor. **43**, 385201(2010).

- **Measurement-induced nonlocality for infinite-dimension**- We define the MiN of ρ by

$$N(\rho) := \sup_{\Pi^A} \|\rho - \Pi^A(\rho)\|_2^2, \tag{4}$$

where the sup is taken over all local von Neumann measurement $\Pi^A = \{\Pi_k^A\}$ that satisfying $\sum_k \Pi_k^A \rho_A \Pi_k^A = \rho_A$. $\|\cdot\|_2$ denotes the Hilbert-Schmidt norm: $\|A\|_2 = [\mathrm{Tr}(A^\dagger A)]^{\frac{1}{2}}$.

- The following properties are straightforward for both finite- and infinite-dimensional cases.

  (i) $N(\rho) = 0$ for any product state $\rho = \rho_A \otimes \rho_B$.

  (ii) $N(\rho)$ is locally unitary invariant, namely, $N[(U \otimes V)\rho(U^\dagger \otimes V^\dagger)] = N(\rho)$ for any unitary operators U and V acting on H_A and H_B, respectively.

  (iii) $N(\rho) > 0$ whenever ρ is entangled since $\Pi^A(\rho)$ is always a classical-quantum state and thus is separable.

  (iv) $0 \leq N(\rho) \leq 4$.

  (v) The MiN of pure state can be easily obtained. Let $|\psi\rangle \in H_A \otimes H_B$ and $|\psi\rangle = \sum_k \lambda_k |k\rangle|k'\rangle$ be its Schmidt decomposition. For the finite-dimensional case, Luo and Fu in [6] showed that $N(|\psi\rangle) = 1 - \sum_k \lambda_k^4$. It is also true for infinite-dimensional case.

- The **quantum discord** for infinite-dimensional systems was firstly discussed in [5].

Let

$$I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho)$$

denote the quantum mutual information of ρ, where $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$ denotes the von Neumann entropy of the state ρ (remark here that $S(\rho)$ maybe $+\infty$). Let $\Pi^A = \{\Pi_k^A = |k\rangle\langle k|\}$ be a local von Neumann measurement. We perform Π^A on ρ, the outcome $\Pi^A(\rho) = \sum_k p_k \rho_k$, where $\rho_k = \frac{1}{p_k}(\Pi_k^A \otimes I_B)\rho(\Pi_k^A \otimes I_B)$ with probability $p_k = \mathrm{Tr}[(\Pi_k^A \otimes I_B)\rho(\Pi_k^A \otimes I_B)]$. Define $I(\rho|\Pi^A) := S(\rho_B) - S(\rho|\Pi^A)$ and $S(\rho|\Pi^A) := \sum_k p_k S(\rho_k)$. The difference

$$D(\rho) := I(\rho) - \sup_{\Pi^A} I(\rho|\Pi^A) \tag{5}$$

is defined to be the quantum discord of ρ, where the sup is taken over all local von Neumann measurement.

- It is proved in [5] that $D(\rho) \geq 0$ for any state $\rho \in \mathcal{S}(H_A \otimes H_B)$. One can check that QD can also be calculated as

$$D(\rho) = I(\rho) - \sup_{\Pi^A} I[\Pi^A(\rho)] \tag{6}$$

for both finite- and infinite dimensional cases. Namely, QD is defined as the infimum of the difference of mutual information of the pre-state ρ and that of the post-state $\Pi^A(\rho)$ with Π^A runs over all local von Neumann measurements.

- For finite-dimensional systems, the CQ states attracted much attention since they can be used for quantum broadcasting [7]. We extend it into infinite-dimensional case via the same scenario.

Classical-quantum state- Similar to Eq.(3), for $\rho \in \mathcal{S}(H_A \otimes H_B)$, $\dim H_A \otimes H_B = +\infty$, if ρ has the following form

$$\rho = \sum_k p_k |k\rangle\langle k| \otimes \rho_k^B, \tag{7}$$

where $\{|k\rangle\}$ is a orthonormal set of H_A, ρ_k^Bs are states of the subsystems B, $p_k \geq 0$ and $\sum_k p_k = 1$, then we call ρ is a classical-quantum state.

[7] S.-L. Luo, *On quantum no-broadcasting*, Lett. Math. Phys. **92**, 143-153(2010).

- Let $\dim H_A \otimes H_B \leq +\infty$,

$$\mathcal{S}_N^0 = \{\rho \in \mathcal{S}(H_A \otimes H_B) : N(\rho) = 0\},$$

$$\mathcal{S}_C = \{\rho \in \mathcal{S}(H_A \otimes H_B) : \rho \text{ is CQ}\},$$

$$\mathcal{S}_D^0 = \{\rho \in \mathcal{S}(H_A \otimes H_B) : D(\rho) = 0\}$$

and $\mathcal{S}_{sep}$ be the set of all separable states acting on $H_A \otimes H_B$. Then

$$\mathcal{S}_N^0 \subseteq \mathcal{S}_C \subseteq \mathcal{S}_D^0 \subseteq \mathcal{S}_{sep}. \tag{8}$$

- It is known that $\mathcal{S}_D^0$ is a zero-measure set [8](that is, each point of this set can be approximated by a sequence of states that not belong to this set with respect to the trace norm) for the finite-dimensional case and $\mathcal{S}_{sep}$ is also a zero-measure set for the infinite-dimensional case [9], thus $\mathcal{S}_N^0$ is a zero-measure set in both finite- and infinite-dimensional cases. (We know now that $\mathcal{S}_D^0$ is also a zero-measure set in infinite-dimensional cases, which answer the question suggested in [5].)

[8] A. Ferraro *et al*, *Almost all quantum states have onnclassical correlations*, Phys. Rev. A **81**, 052318(2010).

[9] R. Clifton and H. Halvorson, *Bipartite mixed states of infinite-dimensional systems are generically nonseparable*, Phys. Rev. A **61**, 012108(1999).

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3 Main result: Nullity of MiN

- In order to state the main result, we need a lemma:
- **Lemma 1.** Let $\dim H_A \otimes H_B \leq +\infty$. Take orthonomal bases $\{|k\rangle\}$ and $\{|i'\rangle\}$ of H_A and H_B, respectively. We write $F_{ij} = |i'\rangle\langle j'|$. Then, for any $\rho \in \mathcal{S}(H_A \otimes H_B)$, we can write ρ as

$$\rho = \sum_{i,j} A_{ij} \otimes F_{ij} \tag{9}$$

where A_{ij}s are trace-class operators acting on H_A and the series converges in the trace norm [10].

[10] Y. Guo and J.-C. Hou, *Comment on "Remarks on the structure of states of composite quantum systems and envariance"[Phys.Lett.A 355(2006)]*, Phys. Lett. A, **375**, 1160-1162(2011).

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It is proved in [11] that, for any density matrix $\rho \in \mathcal{S}(H_A \otimes H_B)$ with $\dim H_A \otimes H_B < +\infty$, if $\rho = \sum_{ij} A_{ij} \otimes F_{ij}$ with A_{ij}s are **mutually commuting** normal matrices, then ρ is separable. In fact, we can prove that such state ρ is not only separable but also a CQ state and that ρ is a CQ state if and only if it admits the form above. Moreover, it can be extended into infinite-dimensional cases:

Theorem 1. Let $\dim H_A \otimes H_B \leq +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. Assume $\rho = \sum_{ij} A_{ij} \otimes F_{ij}$ as in Eq.(9) with respect to some given bases of H_A and H_B. Then ρ is a CQ state if and only if A_{ij}s are **mutually commuting normal operators** acting on H_A.

[11] K.-C. Ha, *Sufficient criterion for separability of bipartite states*, Phys. Rev. A **82**, 014102(2010).

- Theorem 1 implies that QD stems from **noncommutativity** not from **entanglement**.

  We can also find this kind of noncommutativity from another perspective: For finite-dimensional case, it is proved in [8] that if $\rho \in \mathcal{S}_C (= \mathcal{S}_D^0)$ then $[\rho, \rho_A \otimes I_B] = 0$. It is easy to check that it is also valid for infinite-dimensional systems as well:

  Proposition 1. Let $\dim H_A \otimes H_B \leq +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. Then

$$\rho \in \mathcal{S}_C \Rightarrow [\rho, \rho_A \otimes I_B] = 0. \quad (10)$$

- The following is the main result of this talk.

Theorem 2. Let $\dim H_A \otimes H_B \leq +\infty$, $\{|k\rangle\}$ and $\{|i'\rangle\}$ be orthonormal bases of H_A and H_B, respectively, and $\rho \in \mathcal{S}(H_A \otimes H_B)$. Assume that $\rho = \sum_{i,j} A_{ij} \otimes F_{ij} \in \mathcal{S}(H_A \otimes H_B)$ as in Eq.(9) with respect to the given bases. Then $N(\rho) = 0$ if and only if A_{ij}s are **mutually commuting normal operators** and **each eigenspace of ρ_A contained in some eigenspace of A_{ij} for all i and j.**

- Equivalently, Theorem 2 means that $N(\rho) = 0$ if and only if
$$\rho = \sum_k p_k |k\rangle\langle k| \otimes \rho_k^B$$
as in Eq.(7) **with the property that** $\rho_k^B = \rho_l^B$ **whenever** $p_k = p_l$.

- Theorem 2 indicates that the phenomenon of MiN is a manifestation of quantum correlations **due to noncommutativity rather than due to entanglement** as well. And we claim that the **commutativity for a state to have zero MiN is ‘stronger’ than that of zero discordant state**. We illustrate it with the following example.

- **Example.** We consider a $3 \otimes 2$ system. Let

$$\rho = \left(\begin{array}{ccc|ccc} a & \cdot & \cdot & e & \cdot & \cdot \\ \cdot & a & \cdot & \cdot & f & \cdot \\ \cdot & \cdot & b & \cdot & \cdot & g \\ \hline \bar{e} & \cdot & \cdot & c & \cdot & \cdot \\ \cdot & \bar{f} & \cdot & \cdot & c & \cdot \\ \cdot & \cdot & \bar{g} & \cdot & \cdot & d \end{array}\right).$$

(Here, dots denotes the vanished matrix elements.) It is clear that ρ is a CQ state for any positive numbers a, b, c, d and complex numbers e, f, g that make ρ be a state.

However, taking $\Pi^A = \{|\psi_i\rangle\langle\psi_i|\}_{i=1}^3$ with

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, |\psi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

it is easy to see that $\sum_k \Pi_k^A \rho_A \Pi_k^A = \rho_A$ and $\Pi^A(\rho) \neq \rho$ whenever $e \neq f$. If $a + c = b + d$, one can easily conclude that $N(\rho) = 0$ if and only if $a = b$, $c = d$ and $e = f = g$. Hence, there are many CQ states with nonzero MiN.

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- The above example shows that, $\mathcal{S}_N^0$ **is a proper subset of** $\mathcal{S}_D^0$. In addition, $\rho_1, \rho_2 \in \mathcal{S}_N^0$ doesn't imply $\epsilon\rho_1 + (1-\epsilon)\rho_2 \in \mathcal{S}_N^0$ generally, $0 \le \epsilon \le 1$, so $\mathcal{S}_N^0$ is not a convex set. Similarly, $\mathcal{S}_D^0$ (or $\mathcal{S}_C$) is not convex, either.

- From Theorem 2, the following conclusions are clear:

Proposition 2. Let $\dim H_A \otimes H_B \leq +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. Suppose that each eigenspace of ρ_A is of one-dimension and $\rho_A = \sum_k p_k |k\rangle\langle k|$ is the spectral decomposition. Then the local von Neumann measurement Π^A that makes ρ_A invariant is uniquely (up to permutation) induced from $\{|k\rangle\langle k|\}$, and vice versa.

Corollary 1. Let $\dim H_A \otimes H_B \leq +\infty$ and $\rho \in \mathcal{S}_C$. Then $N(\rho) = 0$ provided that each eigenspace of ρ_A is of one-dimension.

Thank you!