The Counterfeit Coin Problems

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1. A Simple Problem

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A Simpler Problem What about 9 coins?
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**An Even Simpler Problem** What about 3 coins?
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**Solution** If there are $3^m$ coins, we need only $m$ weighings.
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**Solution** If there are $3^m$ coins, we need only $m$ weighings.

More generally, if there are $k$ coins with $3^{m-1} < k \leq 3^m$, then we need only $m$ weighings.
<table>
<thead>
<tr>
<th>m</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>1 – 3</td>
<td>4 – 9</td>
<td>10 – 27</td>
<td>28 – 81</td>
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**Deeper Ideas** Tree diagram/graph. Divide and conquer algorithm.
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**Generalization** Suppose you have a three pan balance. Then one can find the fake coin out of $k$ coins by $m$ weighings if $4^{m-1} < k \leq 4^m$. 

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Generalization Suppose you have a three pan balance. Then one can find the fake coin out of \(k\) coins by \(m\) weighings if \(4^{m-1} < k \leq 4^m\).

If there is a \(p\) pan balance, then one can find the fake coin out of \(k\) coins by \(m\) weighings if \((p + 1)^{m-1} < k \leq (p + 1)^m\).
A More Difficult Problem

Suppose 12 coins are given such that one of them has a different weight. Use three weighings to find the different coin, and determine whether it is heavier or lighter.
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More challenging problems

* How many weighings to find a different coin from $k$ given coins.

* What if there are two lighter / different coins?

* What if there are three lighter / different coins?