Matrix inequalities and its applications

Chi-Kwong Li
Department of Mathematics, College of William and Mary,
VA 23187-8795, USA. ckli@math.wm.edu, www.math.wm.edu/~ckli

My plan

* Mathematics is about the study and discovery of patterns arising in practical situations (applications) or intellectual curiosity (theory).

* I will show you some selected results, techniques and research directions in matrix inequalities and their applications. Here are the titles of the 5 parts of my lectures:

  Part 1. Diagonal entries, eigenvalues and singular values.

  Part 2. Submatrices, sum and product of matrices.

  Part 3. Norms and norm inequalities.

  Part 4. Numerical ranges and quantum computing.

  Part 5. Functions and matrices.

* It would be nice if you enjoy these topics. Even if you are not too interested in the subject, hope that you get a glimpse of it so that you can make connections and / or make use of the results and techniques in your study.

* If you can handle the exercises easily, and if you are interested in trying some challenging problems, you are welcomed to join the research group. Of course, you can work on / solve the open problems mentioned in the notes by yourself.

* If you have any comments and suggestions on the material and the lectures, please talk to me or e-mail me so that I can make appropriate adjustment. Thanks.

Notation

$\mathbf{F}$ is the set of real numbers $\mathbf{R}$ or the set of complex numbers $\mathbf{C}$.

$\{e_1, \ldots, e_n\}$ standard basis for $\mathbf{F}^n$, here $\mathbf{F}^n$ is the set of column vectors (or row vectors).

M_n: the set of $n \times n$ matrices.

$\{E_{11}, E_{12}, \ldots, E_{nn}\}$ is the standard basis for M_n.

H_n is the set of complex Hermitian matrices.

Let $A \in M_n$.

$\sigma(A)$ denotes the spectrum of A,

$\rho(A)$ denotes the spectral radius of A,

$W(A)$ denotes the numerical range of A,

$r(A)$ denotes the numerical radius of A,

$s(A) = (s_1(A), \ldots, s_n(A))$ is the vector of singular values of A, where

$s_1(A) \geq \cdots \geq s_n(A) \geq 0$,

$\lambda(A) = (\lambda_1(A), \ldots, \lambda_n(A))$ is a vector of eigenvalues, and

we always assume that $\lambda_1(A) \geq \cdots \geq \lambda_n(A)$ if the eigenvalues are real.

General references

A.R. Amir-Moéz, Extreme Properties of Linear Transformation, Polygonal Publishing House, Washington, N.J., 1990.

R. Bhatia, Matrix Analysis, Springer, New York, 1997.

R. Bhatia, Perturbation Bounds for Matrix Eigenvalues, Classics in Applied Mathematics, SIAM, Philadelphia, 2007.

R. Bhatia, Positive Definite Matrices, Princeton University Press, Princeton, 2007.

L. Hogben (Editor), Handbook in Linear Algebra, Chapman & Hall/CRC, 2007.

R.A. Horn and C.R. Johnson, Matrix Analysis, Cambridge University Press, Cambridge, 1987.

R.A. Horn and C.R. Johnson, Topics in Matrix Analysis, Cambridge University Press, Cambridge, 1991.

M. Marcus and H. Minc, A survey of Matrix Theory and Matrix Inequalities, Dover, New York, 1992.

A.W. Marshall and I. Olkin, Inequalities: Theory of Majorization and its Applications, Academic Press, 1979.

R.C. Thompson, Lecture notes on Matrix Inequalities based on the lectures given at the Johns Hopkins University, 1987.

X. Zhan, Matrix Inequalities, Lecture notes in Mathematics, 1790, Springer, New York, 2002.