Workshop
on
Matrices and Operators

May 24, 2008

T5, Meng Wah Complex

The University of Hong Kong

Morning Session Chaired by Tin-Yau Tam

09:30 – 10:00 Mao-Ting Chien (Soochow University)

The Numerical Radius of Weighted Shift Operators

10:00 – 10:30 Hwa-Long Gau (National Central University)

Numerical Ranges of Reducible Companion Matrices and Nilpotent Operators

Coffee Break

11:00 – 11:30 Raymond Nung-Sing Sze (University of Connecticut)

Elliptical Range Theorems for Generalized Numerical Ranges of Quadratic Operators

11:30 – 12:00 Chi-Kwong Li (College of William and Mary)

Davis-Wielandt Shells of Operators

12:00 – 12:30 Ngai-Ching Wong (National Sun Yat-sen University)

Disjointness Preserving Maps of C^*-Algebras

Afternoon Session Chaired by Chi-Kwong Li

14:00 – 14:30 Che-Man Cheng (University of Macau)

On Pure Imaginary Quaternionic Solutions of the Hurwitz Matrix Equations

14:30 – 15:00 Bit-Shun Tam (Tamkang University)

Maximal Exponents of K-Primitive Matrices: the Polyhedral Cone Case

Coffee Break

15:30 – 16:00 Tin-Yau Tam (Auburn University)

Pfaffian and Decomposable Numerical Range of a Complex Skew Symmetric Matrix

16:00 – 16:30 Yimin Wei (Fudan University)

On Mixed and Componentwise Condition Numbers for Moore-Penrose Inverse and Linear Least Square Problems

Titles and Abstracts

On Pure Imaginary Quaternionic Solutions of the Hurwitz Matrix Equations

Speaker Che-Man Cheng (University of Macau), fstcmc@umac.mo

Co-author Yik-Hoi Au-Yeung (University of Hong Kong)

Abstract

In this talk, we discuss the maximum number of $n \times n$ pure imaginary quaternionic solutions to the Hurwitz matrix equations given by

$$T_i T_j^* + T_j T_i^* = 2\delta_{ij} I, \quad i, j = 1, \ldots, p,$$

where δ_{ij} is the Kronecker delta.

The Numerical Radius of Weighted Shift Operators

Speaker Mao-Ting Chien (Soochow University), mtchien@scu.edu.tw

Co-author Hiroshi Nakazato (Hirosaki University)

Abstract

Let T be a bounded linear operator on a complex Hilbert space H. For $0 \leq q \leq 1$, the q-numerical range $W_q(T)$ of T is defined by

$$W_q(T) = \{\langle T\xi, \eta\rangle : ||\xi|| = ||\eta|| = 1, \langle \xi, \eta\rangle = q\}.$$

$W_q(T)$ is a bounded convex subset of $\mathbf{C}$. Its q-numerical radius is denoted by

$$w_q(T) = \sup\{|z| : z \in W_q(T)\}.$$

If T is a weighted shift operator on $\ell^2(\mathbf{N})$ with bounded weights $\{s_n\}$, it is known that $W_q(T)$ is a circular disk about the origin. This paper deals with computations of the the q-numerical radius of weighted shift operators with geometric weights and periodic weights.

Numerical Ranges of Reducible Companion Matrices and Nilpotent Operators

Speaker Hwa-Long Gau (National Central University), hlgau@math.ncu.edu.tw

Abstract

In this talk, we will present some results on the numerical ranges of reducible companion matrices and nilpotent operators. Recall that an n-by-n complex matrix B is said to be of class S_n^{-1} if all eigenvalues of B have modulus greater than one, and B satisfies rank$(I_n - B^*B) = 1$. We prove that if B_1 and B_2 are S_n^{-1}-matrices, then $W(B_1) = W(B_2)$ if and only if B_1 and B_2 are unitarily equivalent. Using this result, we can prove that if C_1 and C_2 are n-by-n reducible companion matrices, then $W(C_1) = W(C_2)$ if and only if $C_1 = C_2$.

For any operator A on a complex Hilbert space, let $w(A)$ and $w_0(A)$ denote its numerical radius and the distance from the origin to the boundary of its numerical range, respectively. We prove that if $A^n = 0$, then $w(A) \leq (n-1)w_0(A)$, and, moreover, if A attains its numerical radius, then the following are equivalent: (1) $w(A) = (n-1)w_0(A)$, (2) A is unitarily equivalent to an operator of the form $aA_n \oplus A'$, where a is a scalar satisfying $|a| = 2w_0(A)$, A_n is the n-by-n matrix

$$\begin{bmatrix} 0 & 1 & \cdots & 1 \\ & 0 & \ddots & \vdots \\ & & \ddots & 1 \\ & & & 0 \end{bmatrix},$$

and A' is some other operator, and (3) $W(A) = bW(A_n)$ for some scalar b.

Davis-Wielandt Shells of Operators

Speaker Chi-Kwong Li (College of William and Mary), ckli@math.wm.edu

Co-authors Yiu-Tung Poon (Iowa State University) and Nung-Sing Sze (University of Connecticut)

Abstract

Basic properties of Davis-Wielandt shells are presented. Conditions on two operators A and B with the same Davis-Wielandt shells are analyzed. Special attention is given to the case when B is a compression of A, and when $B = A^*$, A^t, or $(A^*)^t$, where A^t is the transpose of A with respect to an orthonormal basis. The results are used to study the point spectrum, approximate spectrum, and residual spectrum of the sum of two operators. Relation between the geometrical properties of the Davis-Wielandt shells and algebraic properties of operators are obtained. Complete descriptions of the Davis-Wielandt shells are given for several classes of operators.

Elliptical Range Theorems for Generalized Numerical Ranges of Quadratic Operators

Speaker Raymond Nung-Sing Sze (University of Connecticut), sze@math.uconn.edu

Co-authors Chi-Kwong Li (College of William and Mary) and Yiu-Tung Poon (Iowa State University)

Abstract

It is well known that the classical numerical range of a quadratic operator is an elliptical disk. In this talk, results for different kinds of generalized numerical ranges of a quadratic operator will be presented. In particular, for a given quadratic operator, the rank-k numerical range, the essential numerical range, and the q-numerical range are elliptical disks; the c-numerical range is a sum of elliptical disks, and the Davis-Wielandt shell is an ellipsoid with or without interior.

Maximal Exponents of K-Primitive Matrices: the Polyhedral Cone Case

Speaker Bit-Shun Tam (Tamkang University), bsm01@mail.tku.edu.tw.

Coauthor Raphael Loewy (Technion)

Abstract

Let K be a proper (i.e., closed, pointed, full convex) cone in R^n. An $n \times n$ matrix A is said to be K-primitive if there exists a positive integer k such that $A^k(K \setminus \{0\}) \subseteq \text{int}\, K$; the least such k is referred to as the exponent of A and is denoted by $\gamma(A)$. For a polyhedral proper cone K, the maximum value of $\gamma(A)$, taken over all K-primitive matrices A, is denoted by $\gamma(K)$. It is proved that for any positive integers $m, n, 3 \leq n \leq m$, the maximum value of $\gamma(K)$, as K runs through all n-dimensional polyhedral cones with m extreme rays, equals $(n-1)(m-1)+1$ when m is even or m and n are both odd, and is at least $(n-1)(m-1)$ when m is odd and n is even. For the cases when $m = n, m = n+1$ or $n = 3$, the cones K and the corresponding K-primitive matrices A such that $\gamma(K)$ and $\gamma(A)$ attain the maximum value are identified up to linear isomorphism and cone-equivalence respectively.

Pfaffian and Decomposable Numerical Range of a Complex Skew Symmetric Matrix

Speaker Tin-Yau Tam (Auburn University), tamtiny@auburn.edu

Abstract

The decomposable numerical range

$$R_k^{\wedge}(A) := \{\det((O^T AO)[k|k]) : O \in SO(n)\}$$

and the Pfaffian numerical range

$$P_{2k}(A) := \{\mathrm{Pf}\ (O^T AO[2k|2k]) : O \in SO_n\}$$

of a complex skew symmetric matrix A are studied. Here

$$\mathrm{Pf}\ (B) = \frac{1}{2^n n!} \sum_{\sigma \in S_{2n}} \mathrm{sgn}(\sigma) \prod_{i=1}^{n} b_{\sigma(2i-1),\sigma(2i)}$$

is the Pfaffian of a complex skew symmetric matrix B of size $2n \times 2n$. Some star-shapedness and convexity results are obtained.

On Mixed and Componentwise Condition Numbers for Moore-Penrose Inverse and Linear Least Square Problems

Speaker Yimin Wei (Fudan University), ymwei_cn@yahoo.com

Abstract

Classical condition numbers are normwise: they measure the size of both input perturbations and output errors using some norms. To take into account the relative of each data component, and, a possible data sparseness, componentwise condition numbers have been increasingly considered. These are mostly of two kinds: mixed and componentwise. In this talk, we give explicit expressions, computable from the data, for the mixed and componentwise condition numbers for the computation of the Moore-Penrose inverse as well as for the computation of solutions and residues of linear least squares problems. In both cases the data matrices have full column (row) rank.

Disjointness Preserving Maps of C^*-Algebras

Speaker Ngai-Ching Wong (National Sun Yat-sen University), wong@math.nsysu.edu.tw

Abstract

There are at least four versions to say a linear map T between two $C*$-algebras is 'disjointness preserving':

$ab = 0$, $a^*b = 0$, $ab^* = 0$ or $a^*b = ab^* = 0$ implies $TaTb = 0$, $(Ta)^*(Tb) = 0$, $(Ta)(Tb)^* = 0$ or $(Ta)^*(Tb) = (Ta)(Tb)^* = 0$, respectively.

The second, third and the last say T send operators with orthogonal ranges, orthogonal domains, or both, to operators with the same disjointness. The first, however, lacks of such trivial geometrical sense.

Quite an amount of efforts has been put in characterizing such disjointness preserving maps in recent years. They are basically a homomorphism or a $*$-homomorphism followed by a multiplication of a left or right or central multiplier.

In this talk, I will give a brief report on these results.