Workshop
on
Matrices and Operators

July 13, 2007

T5, Meng Wah Complex

The University of Hong Kong

Morning Session (Chaired by Yiu-Tung Poon)

09:00 – 09:45 Bit-Shun Tam, Tamkang University
Exponents of K-primitive matrices

09:45 – 10:30 Chi-Kwong Li, College of William and Mary
G-invariant norms and bicircular projections

Break

10:45 – 11:30 Raymond Nung-Sing Sze, University of Connecticut
The "best" conditional numbers for Markov chain

11:30 – 12:15 Hwa-Long Gau, National Central University
Numerical ranges of reducible companion matrices

Lunch

Afternoon Session (Chaired by Chi-Kwong Li)

14:00 – 14:45 Pei Yuan Wu, National Chiao Tung University
Sums of orthogonal projections

14:45 – 15:30 Ngai-Ching Wong, National Sun Yat-sen University
Invertibility of linear sums of two k-potents and nilpotents

Break

15:45 – 16:30 Yiu-Tung Poon, Iowa State University
Eigenvalues of the sum of operators from unitary similarity orbits

Numerical ranges of reducible companion matrices

Hwa-Long Gau
National Central University
hlgau@math.ncu.edu.tw

Abstract

In this talk, we give a criterion for a companion matrix to be reducible and give a complete characterization of reducible companion matrices. We also consider properties of the numerical ranges of reducible companion matrices. To be more precise, if C is an n-by-n reducible companion matrix which is unitarily equivalent to the direct sum of A and B, we study properties of the numerical ranges of these direct summands A and B. We show that the boundary of such a numerical range is a differentiable algebraic curve. We also obtain a criterion for the equality $W(C) = W(A)$.

G-invariant norms and bicircular projections

Chi-Kwong Li
College of William and Mary
ckli@math.wm.edu

Abstract

It is shown that for many finite dimensional normed vector spaces $\mathbf{V}$ over $\mathbf{C}$, a linear projection $P : \mathbf{V} \to \mathbf{V}$ will have nice structure if $P + \lambda(I - P)$ is an isometry for some complex unit not equal to one. From these results, one can readily determine the structure of bicircular projections, i.e., those linear projections P such that $P+\mu(I-P)$ is a an isometry for every complex unit μ. The key ingredient in the proofs is the knowledge of the isometry group of the given norm. The proof techniques also apply to real vector spaces. In such cases, characterizations are given to linear projections P such that $P - (I - P) = 2P - I$ is an isometry.

This is based on a joint paper with Maja Fošner (University of Maribor, Slovenia), and Dijana Ilišević (University of Zagreb, Croatia).

Eigenvalues of the sum of operators from unitary similarity orbits

Yiu-Tung Poon
Iowa State University
ytpoon@iastate.edu
Joint work with Chi-Kwong Li and Nung-Sing Sze

Abstract

Let A and B be $n \times n$ complex matrices. Characterization is given for the set $\mathcal{E}(A, B)$ of eigenvalues of matrices of the form $U^*AU + V^*BV$ for some unitary matrices U and V. Consequences of the results are discussed and computer algorithms and programs are designed to generate the set $\mathcal{E}(A, B)$. The results refine those of Wielandt on normal matrices. Extensions of the results to the sum of operators on an infinite dimensional Hilbert space and sum of three or more unitary similarity orbits are also considered.

The "best" condition number for Markov chains

Raymond Nung-Sing Sze
University of Connecticut
sze@math.uconn.edu

Abstract

For an irreducible stochastic matrix T, we studied a class of condition numbers $\kappa(T)$, which measures the stability of the corresponding stationary distribution when T is perturbed. The "best" condition numbers are investigated and the best lower bounds for these condition numbers are given. Finally, the class of matrices that attain the best lower bound is discussed.

This talk is based on a joint work with M. Neumann.

Exponents of K-primitive matrices

Bit-Shun Tam
Tamkang University
bsm01@mail.tku.edu.tw
Joint work with Raphael Loewy (Technion, Israel)

Abstract

Let K be a proper (i.e., closed, pointed, full convex) cone in R^n. An $n \times n$ matrix A is said to be K-primitive if there exists a positive integer k such that $A^k(K \setminus \{0\}) \subseteq \text{int } K$; the least such k is referred to as the exponent of A and is denoted by $\gamma(A)$. For a polyhedral proper cone K, the maximum value of $\gamma(A)$, taken over all K-primitive matrices A, is denoted by $\gamma(K)$. We treat the problem of determining the maximum value of $\gamma(K)$ as K runs through all n-dimensional polyhedral cones with m extreme rays and solve for it three special cases, namely, when $m = n + 1$ and when $n = 3$. In each case, we determine also the cones K in the relevant class (and the corresponding K-primitive matrices A) such that $\gamma(K)$ (and $\gamma(A)$) attain the maximum value. Some partial results for the general case are also obtained.

Invertibility of linear sums of two k-potents and nilpotents

Ngai-Ching Wong
National Sun Yat-sen University
wong@math.nsysu.edu.tw

Abstract

Let P and Q be two idempotents in a Banach algebra. It is known only quite recently that $P+Q$ is invertible if and only if $aP+bQ$ is invertible for any, and thus all, scalars a, b with ab and $a+b$ being nonzero. We show by counter examples this result cannot be extended to k-potents with $k>2$, or partial isometries. However, we show that the result holds for the case of square zero elements, but not for any other kind of nilpotent.

Sums of orthogonal projections

Pei Yuan Wu
National Chiao Tung University
pywu@math.nctu.edu.tw

Abstract

In this talk, we address the problem: Which (bounded linear) operator T on a complex separable Hilbert space H can be expressed as the sum of finitely many (orthogonal) projections? When H is finite-dimensional, a complete characterization was obtained by P. A. Fillmore in 1969. We will try to see how his conditions can be suitably generalized to the infinite-dimensional case. We are only partially successful in this endeavor. This has been an on-going research project with Man-Duen Choi since 1988.