Schedule, Titles, and Abstracts of the Tenth Numerical Range Workshop

Uniwersytet Jagielloński, Collegium Maius, ul. Jagiellońska 15, Kraków.

June 27 (Sunday) - June 29 (Tuesday), 2010.

organized by

Instytut Matematyki, Uniwersytet Jagielloński
with collaboration and financial support of
Instytut Matematyczny, Polska Akademia Nauk

Endorsed by the International Linear Algebra Society

Schedule (Tentative)

July 27 (Sunday) Arrival and sightseeing in Krakow.

July 28 (Monday)

9:00 - 10:30 a.m. Chair: Chi-Kwong Li

Man-Duen Choi, Numerical ranges and dilations.

Hiroshi Nakazato, Kepler's vicarious ovum lies on an algebraic curve with degree 12, genus 7.

Mao-Ting Chien, Numerical range for orbits under a central force.

10:30 - 11:00 a.m. coffee break.

11:00 a.m. - 12:30 p.m. Chair: Yiu-Tung Poon

Karol Zyczkowski, On restricted numerical range.

Zbigniew Puchala, Numerical shadow and its generalizations.

Frank Uhlig, Methods for the inverse numerical range problem

12:30- 2:00 p.m. **Workshop photo** and Lunch break.

2:00 - 3:30 p.m. Chair: Jaroslav Zemanek

Yiu-Tung Poon, Spectrum, numerical range and Davis Wielandt shell of a normal operator.

Chi-Kwong Li, Spectra, norms, and numerical ranges of generalized quadratic operators.

Dariusz Cichon, Local spectral radius for unbounded operators.

3:30 - 4:00 p.m. Coffee break

4:00 - 5:30 p.m. Chair: Tin-Yau Tam

Michel Crouzeix, A matrix factorization based on numerical radii.

Grzegorz Lewicki, Best approximation in numerical radius.

Javier Meri, On the numerical radius of some classes of operators in L_p spaces.

6:15 - 7:30 p.m. visit to University Museum, Collegium Maius

7:30 p.m. - **Workshop dinner**

June 29 (Tuesday)

9:00 - 10:30 a.m. Chair: Man-Duen Choi

Miroslav Fiedler, Complementary basic matrices.

Moshe Goldberg, Homotonic algebras

Fuzhen Zhang, Ky Fan: Beautiful results and beautiful life

10:30 - 11:00 a.m. Coffee break

11:00 a.m. - 12:30 p.m. Chair: Thomas Schulte-Herbrüggen

Iwona Wróbel, On the Gauss-Lucas theorem and the numerical range of companion matrices

Alexander Markus, Connectedness of zero sets of some functionals and convexity of C-numerical ranges.

Tin-Yau Tam, A cousin of the numerical range.

12:30 - 2:00 p.m. Lunch break

2:00 - 3:30 p.m. Chair: Karol Zyczkowski

Thomas Schulte-Herbrüggen, Least-squares approximation by matrix orbits and rank-k numerical ranges: Flows for optimization in quantum dynamics.

Raymond Nung-Sing Sze, Higher rank numerical range of normal matrices.

Natalia Bebiano, Higher rank numerical range for J-Hermitian matrices.

3:30 - 4:00 p.m. Coffee break

4:00 - 5:30 p.m. Chair: Tom Laffey

Aikaterini Aretaki, The higher rank numerical range of matrix polynomials.

K.-H. Förster, The block numerical range of matrix polynomials.

A. Nata, On the boundary of the Krein space tracial numerical range.

Title The higher rank numerical range of matrix polynomials

Speaker Aikaterini Aretaki, National Technical University of Athens, Greece

Co-author John Maroulas, National Technical Univ. of Athens, Greece, maroulas@math.ntua.gr

Abstract

Let $\mathcal{M}_n(\mathbf{C})$ be the algebra of $n \times n$ complex matrices. For matrix polynomials $L(\lambda) = A_m\lambda^m + \ldots + A_1\lambda + A_0$ with $A_i \in \mathcal{M}_n(\mathbf{C})$, $i = 0, \ldots, m$, the notion of the higher rank numerical range $\Lambda_k(L(\lambda))$ is introduced here and it is described as an intersection of the numerical ranges $w(M^*L(\lambda)M)$ through all $(n-k+1)$-dimensional compressions $M^*L(\lambda)M$ of $L(\lambda)$. Further, the boundedness and connectedness of $\Lambda_k(L(\lambda))$ are investigated. The sharp points of $\Lambda_k(L(\lambda))$ are defined and their relation to $w(L(\lambda))$ is presented as well as a relationship between $\Lambda_k(L(\lambda))$ and $\Lambda_k(C_L(\lambda))$, where $C_L(\lambda)$ is the companion polynomial of $L(\lambda)$. Finally, the boundary points of the joint higher rank numerical range $\Lambda_k(\mathbf{A})$ of an $(m+1)$-tuple $\mathbf{A} = (A_0, \ldots, A_m)$ are considered with respect to the boundary points of the joint numerical range $w(\mathbf{A})$ and an interplay of $\Lambda_k(L(\lambda))$ and $\Lambda_k(\mathbf{A})$ is discussed.

Title Higher Rank Numerical Ranges for J-Hermitian Matrices

Speaker Natalia Bebiano, University of Coimbra.

Abstract

We consider indefinite higher-rank versions of the classical numerical range for matrices. Our results are stated to J-Hermitian matrices, $J = I_r \oplus -I_{n-r}$, $0 < r < n$, that is, Hermitian matrices on an indefinite inner product space. Particular attention is paid to aspects of the theory that parallel the case of linear operators on Hilbert spaces, which has deserved special attention due to its intimate relation with the problem of error correcting in quantum computing.

Title Numerical range for orbits under a central force

Speaker Mao-Ting Chien, Soochow University, Taiwan, mtchien@scu.edu.tw

Co-author Hiroshi Nakazato, Hirosaki University, Japan, nakahr@cc.hirosaki-u.ac.jp

Abstract

We present an explicit form for the central force that describes the orbit of a roulette curve, and interpret the orbit of the roulette curve as an algebraic curve $F(1, x, y) = 0$ associated to the homogeneous polynomial $F(t, x, y) = \det(tI_n + x(A + A^*)/2 + y(A - A^*)/(2i))$ of a matrix A. The hodograph of the orbit is obtained as the boundary generating curve of the numerical range of A.

Title Numerical ranges and dilations

Speaker Man-Duen Choi, Department of Mathematics, University of Toronto

Abstract

This is a survey talk on the relation between numerical range inclusions and dilations. Namely, if a $k \times k$ complex matrix A is the top left corner of an $n \times n$ complex matrix B, then the numerical range inclusion holds. Conversely, if the numerical range of A is subset of the numerical range of B, it will be intriguing to get some sort of constructional features relating both matrices.

Title Local spectral radius for unbounded operators

Speaker Dariusz Cichon, Jagiellonian University, Dariusz.Cichon@im.uj.edu.pl

Co-authors Il Bong Jung (Kyungpook National University) and Jan Stochel (Uniwersytet Jagiellonski).

Abstract

Focusing on local spectral radius of an unbounded operator we distinguish related linear subspaces composed of vectors whose spectral radius is no greater than a fixed nonnegative parameter. The subspaces are invariant under the operator in question thus considering family of associated restrictions we define localoid and locally normaloid opereators (taking into account spectral radii and norms of restrictions, respectively). The question arises how far those operators are from being normal? The answer to this will be preceded by results and examples elucidating the behaviour of local spectral radii for unbounded operators.

Title A matrix factorization based on numerical radii

Speaker Michel Crouzeix, Université de Rennes 1, michel.crouzeix@univ-rennes1.fr

Abstract

We described, in the matrix case, a variant of a decomposition introduced by M.A. Dritschel and H.J. Woederman. In particular this shows that each $n \times n$ matrix A may be written on the form

$$A = 2 \sin B \, U^* \mathrm{diag}\,(a_1, \ldots, a_n) \, U \cos B,$$

with B a self-adjoint matrix satisfying $0 \leq B \leq \frac{\pi}{2}$, U a unitary matrix, and $a_1, \ldots, a_n \in W(A)$. This form is a variant of a Andô decomposition.

Complementary basic matrices

Speaker Miroslav Fiedler, Inst. of Computer Science, Academy of Sciences of the Czech Republic, Pod vodáren. věží 2, 182 07 Praha 8, Czech Republic, (fiedler@cs.cas.cz).

Abstract

In a few papers [1] - [3], the class of complementary basic matrices (CB-matrices) was introduced as matrices, if of order n, $A = G_{i_1} G_{i_2} \cdots G_{i_{n-1}}$, where $(i_1, i_2, \ldots, i_{n-1})$ is a permutation of $(1, 2, \ldots, n-1)$, and the matrices G_k, $k = 1, \ldots, n-1$ have the form

$$G_k = \begin{bmatrix} I_{k-1} & & \\ & C_k & \\ & & I_{n-k-1} \end{bmatrix}$$

for some 2×2 matrices C_k. It was observed that 1. independently of the permutation, all such matrices with given C_i's have the same spectrum (though they do not form a similarity class), 2. the classical companion matrix belongs to the class of CB-matrices, 3. the multiplication of the G_i's is intrinsic (cf. [4]) in the sense that in every product of a row and a column there is at most one term different from zero, i.e. each non-zero entry of A is the product of some of the entries of the matrices C_i; there is no addition. We add some more properties of CB-matrices and pose a problem to use the property 2., the numerical range and the Gershgorin circles results to obtain estimates of the roots of a univariate polynomial equation.

References

[1] M. Fiedler, Complementary basic matrices. *Linear Algebra Appl.* 384(2004), 199 – 206.

[2] M. Fiedler, A note on companion matrices. *Linear Algebra Appl.* 372(2003), 325 - 331.

[3] M. Fiedler, A note on sign-nonsingular matrices. *Linear Algebra Appl.* 408(2005), 14 - 18.

[4] M. Fiedler, Intrinsic products and factorizations of matrices. *Linear Algebra Appl.* 428(2008), 5 - 13.

Title The Block Numerical Range of Matrix Polynomials

Speaker K.-H. Förster, Technical University of Berlin.

Co-authors N. Hartanto, M.M. Nafalska and B. Nagy.

Abstract

The block numerical range of a linear operator in Hilbert spaces and of a monic matrix polynomial was introduced by Ch. Tretter and M. Wagenhofer (2003); the spectrum, the usual numerical range and the quadratic numerical range are special cases.

We discuss the relation between the block numerical range of arbitrary matrix polynomials and its linearizations or degree reductions. Properties of the block numerical range of semi-monic Perron-Frobenius polynomials will be considered in detail.

Title Homotonic Algebras

Speaker Moshe Goldberg, Technion – Israel Institute of Technology, Haifa, Israel, goldberg@math.technion.ac.il.

Abstract

An algebra $\mathcal{A}$ of real or complex valued functions defined on a set $\mathbf{T}$ shall be called *homotonic* if $\mathcal{A}$ is closed under forming of absolute values, and for all f and g in $\mathcal{A}$, the product $f \times g$ satisfies $|f \times g| \leq |f| \times |g|$. Our main purpose in this talk is two-fold: To show that the above definition is equivalent to an earlier definition of homotonicity, and to provide a simple inequality which characterizes sub-multiplicativity and strong stability for weighted sup norms on homotonic algebras.

Title Best approximation in numerical radius

Speaker Grzegorz Lewicki, Department of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland, Grzegorz.Lewicki at im.uj.edu.pl

Co-authors Asuman Güven Aksoy

Abstract

Let X be a reflexive Banach space. In this paper we give a necessary and sufficient condition for an operator $T \in \mathcal{K}(X)$ to have the best approximation in numerical radius from the convex subset $\mathcal{U} \subset \mathcal{K}(X)$, where $\mathcal{K}(X)$ denotes the set of all linear, compact operators from X into X. We will also present an application to minimal extensions with respect to the numerical radius. In particular some results on best approximation in norm will be generalized to the case of the numerical radius.

Title Spectra, norms and numerical ranges of generalized quadratic operators

Speaker Chi-Kwong Li, College of William and Mary.

Co-author Masaru Tominaga, Hiroshima Institute of Technology; Yiu-Tung Poon, Iowa State University.

Abstract

A bounded linear operator acting on a Hilbert space is a generalized quadratic operator if it has an operator matrix of the form

$$\begin{pmatrix} aI & cT \\ dT^* & bI \end{pmatrix}.$$

It reduces to a quadratic operator if $d = 0$. In this paper, spectra, norms, and various kinds of numerical ranges of generalized quadratic operators are determined. Some operator inequalities are also obtained. In particular, it is shown that for a given generalized quadratic operator, the rank-k numerical range, the essential numerical range, and the q-numerical range are elliptical disks; the c-numerical range is a sum of elliptical disks. The Davis-Wielandt shell is the convex hull of a family of ellipsoids unless the underlying Hilbert space has dimension 2.

Title Connectedness of zero sets of some functionals and convexity of C-numerical ranges

Speaker A. Markus, Ben-Gurion University of the Negev, Israel (markus@math.bgu.ac.il).

Abstract

The first proof of the convexity of numerical range given by Hausdorff was based on an auxiliary statement on connectedness of zero sets of Hermitian forms (see [1], pp. 314-315). This approach was systematically used for joint numerical ranges in [2,3], and we follow a similar way. We prove, in particular, the following statement.

Theorem 1. Let H be a Hilbert space, $A = A^* \in \mathcal{L}(H), n \in \mathbf{N}, \{c_k\}_1^n \subset \mathbf{R}$. Then the set of all orthonormal systems $\{e_k\}_1^n$ such that $\sum_{k=1}^n c_k(Ae_k, e_k) = 0$, is connected.

We show that Theorem 1 implies some convexity results for C-numerical ranges, in particular, the Westwick theorem (for details see [4], Section 5.5). Some other connectedness results and their applications also will be presented.

References

[1] P.R. Halmos, *A Hilbert Space Problem Book.* Springer-Verlag, New York, 1982.

[2] Yu. Lyubich, A. Markus, *Connectivity of level sets of quadratic forms and Hausdorff-Toeplitz type theorems.* Positivity **1** (1997), 239-254.

[3] P. Binding, A. Markus, *Joint zero sets and ranges of several Hermitian forms over complex and quaternionic scalars.* Linear Algebra Appl. **385** (2004), 63-72.

[4] K.E. Gustafson, D.K.M. Rao, *Numerical Range.* Springer-Verlag, Berlin, 1997.

Title On the numerical radius of some classes of operators in L_p spaces

Speaker Javier Merí, Universidad de Granada (Spain), jmeri@ugr.es

Co-authors M. Martín, Universidad de Granada (Spain); M. Popov, Chernivtsi National University (Ukraine); B. Randranantoanina, Miami University (USA)

Abstract

We introduce the concept of absolute numerical radius for bounded linear operators defined on L_p spaces. This allows us to show that the numerical radius is an equivalent norm to the usual operator norm for the real L_p spaces. Besides, we give lower bounds for the numerical radius of rank-one operators and finite-rank operators on the space $L_p[0, 1]$ in both the real and complex setting.

Title Kepler's vicarious ovum lies on an algebraic curve with degree 12, genus 7

Speaker Hiroshi Nakazato, Hirosaki University, (nakahr@cc.hirosaki-u.ac.jp)

Abstract

In "Astronomia Nova" Johannes Kepler provided some convex curves for models of planetary orbits. Some of his models are related to the numerical range of matrices. In this talk I discuss the algebraic properties of his models.

Title On the boundary of the Krein space tracial numerical range

Speaker A. Nata, CMUC and Department of Mathematics of the Polytechnic Institute of Tomar, Portugal, and anata@ipt.pt

Co-authors N. Bebiano (CMUC and Department of Mathematics, University of Coimbra, Portugal), H. Nakazato (Hirosaki University, Department of Mathematical Sciences, Japan) and J. P. da Providência (Department of Physics, University of Coimbra, Portugal)

Abstract

In this talk, tracial numerical ranges, $W_C^J(A)$, associated with matrices in an indefinite inner product space are investigated. The equations of the boundary generating curve of $W_C^J(A)$ are obtained and the connection between the J-normality of A and the smoothness of $W_C^J(A)$ is investigated. Namely, if A is J-normal, a characterization of $W_C^J(A)$ is deduced. As an application, a numerical algorithm for plotting the tracial numerical range of an arbitrary complex matrix, is presented. Our approach uses the elementary idea that the boundary may be traced by computing the supporting lines.

Title Spectrum, Numerical Range and Davis-Wielandt Shell of a Normal Operator

Speaker Yiu-Tung Poon, Iowa State University.

Co-author Chi-Kwong Li, William and Mary.

Abstract

It is well known that the closure of the numerical range of a normal operator is the convex hull of its spectrum. Thus, the two sets have the same interior. But the boundary structure of the numerical range is not so well understood. In this paper, characterization is given to points of the numerical range that lie on the boundary. It is shown that such boundary points reveal a lot of information about the normal operator that cannot be obtained from its spectrum and point spectrum. For instance, such a boundary point always associates with an invariant (reducing) subspace of the normal operator. It follows that a normal operator acting on a separable Hilbert space cannot have a closed strictly convex set as its numerical range. Similar results are obtained for the Davis-Wielandt shell of a normal operator. One can deduce additional information of the normal operator by studying the boundary of its Davis-Wielandt shell.

Title Numerical Shadow and its generalizations

Speaker Zbigniew Puchala, Polish Academy of Sciences, Gliwice.

Co-authors C. Dunkl, P. Gawron, J. Holbrook, J.A. Miszczak, K. Zyczkowski

Abstract

We analyze probability distributions induced on the numerical range of a given operator by the unique unitarily invariant (Fubini-Study) measure on the set unit vectors. Such a density will be called 'numerical shadow of an operator', since for a normal operator it covers the numerical range with the probability corresponding to the projection of a regular N-simplex embedded in N-1dimensions into a plane.

As the numerical range of a non-normal matrix is not a polygon, the corresponding numerical shadow occurs to be a more complicated probability distribution. For instance, the shadow of a non-normal operator of size N=2 corresponds to the shadow of a hollow sphere S^2 projected onto the plane. We derive explicit results for the shadow of certain non-normal operators of small sizes and analyze numerical shadow of the Jordan nilpotent J_N.

Title Least-Squares Approximation by Matrix Orbits and Rank-k Numerical Ranges: Flows for Optimisation in Quantum Dynamics

Speaker T. Schulte-Herbrüggen, Technical University of Munich (TUM), tosh@ch.tum.de

Co-authors C.-K. Li, College of William and Mary, Williamsburg VA, USA, Y.-T. Poon, Iowa State University, Ames IA, USA, N.-S. Sze, Hong-Kong Polytechnic University, Hong Kong.

Abstract

For the first set of problems, let (A) denote the orbit of a complex or real matrix A under a certain equivalence relation such as unitary similarity, unitary equivalence, etc. Based on the differential geometry of the various orbits seen as Riemannian manifolds with bi-invariant metrics on their respective tangent spaces, efficient gradient-flow algorithms are constructed to determine the best approximation of a given matrix A_0 by the sum of matrices in $S(A_1), \ldots, S(A_N)$ in the sense of finding the Euclidean least-squares distance

$$\min\{\|X_1 + \cdots + X_N - A_0\| : X_j \in S(A_j),\ j = 1, \ldots, N\}.$$

For the second set of problems, let H_n denote the set of $n \times n$ Hermitian matrices. Suppose $1 \leq k \leq (n-1)/2$ so $\mathcal{P}_k$ is the set of rank-k projectors. Then the rank k-numerical range of $\mathbf{A} = (A_1, \ldots, A_m) \in H_n^m$ is defined as the set

$$\Lambda_k(\mathbf{A}) = \{(a_1, \ldots, a_m) \in \mathbf{R}^m : \exists P \in \mathcal{P}_k \text{ with } PA_jP = a_jP \text{ for all } j \in [1, m]\}.$$

We are interested in gradient-flow based computer programs to generate (or approximate) $\Lambda_k(\mathbf{A})$ or checking whether $\Lambda_k(\mathbf{A})$ is empty. — The basic idea takes $\varepsilon > 0$ as a given a tolerance. For each $P \in \mathcal{P}_k$, we check whether

$$|PA_jP - a_jP| < \varepsilon \qquad \text{for } j = 1, \ldots, m.$$

If yes, then $(a_1, \ldots, a_m)$ is a point in $\Lambda_k^\varepsilon(\mathbf{A})$.

Both problems are addressed by designing gradient-flows. We discuss their differential geometry in view our special focus on applications in quantum dynamics of open systems.

Title Higher Rank Numerical Range of Normal Matrices

Speaker Raymond Nung-Sing Sze, Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Hong Kong (raymond.sze@inet.polyu.edu.hk).

Co-authors H.L. Gau (Central University), C.K. Li (College of William and Mary), and Y.T. Poon (Iowa State University).

Abstract

It is known that the higher rank numerical range $\Lambda_k(A)$ of a normal matrix A is either an empty set or a convex polygon in $\mathbf{C}$. In this talk, the following two interesting problems will be addressed.

1. Given a normal matrix A, determine the number of sides of the convex polygon $\Lambda_k(A)$.

2. Given a polygon $\mathcal{P}$ and $k > 1$, construct a normal matrix A with smallest dimension such that $\Lambda_k(A) = \mathcal{P}$.

Title A cousin of the numerical range

Speaker Tin-Yau Tam, Auburn University. (tamtiny@auburn.edu)

Abstract

Abstract: We study the range

$$S(A) := \{x^T Ay : x, y \text{are two columns of an } n \times n \text{ orthogonal matrix}\}$$

where A is an $n \times n$ complex skew symmetric matrix. When $n = 3, 4, 5, 6$, relation between $S(A)$ and the classical numerical range $W(A)$ is given. We also obtain the axiomatic characterization of $S(A)$, the characterization of sharp points and extreme points of $S(A)$ and power inequality for the radius.

Title Methods for the Inverse Numerical Range Problem

Speaker Frank Uhlig, Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849-5310, USA (uhligfd@auburn.edu)

Co-authors Christos Chorianopoulos and Panayiotis Psarrakos, Department of Mathematics, National Technical University of Athens, Zografou Campus, 15780 Athens, Greece (horjoe@yahoo.gr and ppsarr@math.ntua.gr).

Abstract

For a complex square matrix A and a given point $\mu \in C$, the inverse field of values problem consists of finding a unit generating vector $x \in C^n$ with $x^*Ax = \mu$ if possible and deciding whether the given μ lies inside or outside the fields of values of A.
The problem was introduced at WONRA8 in Bremen in 2006 and a randomized search algorithm for finding such x explained by the presenter there. In 2009, a nearly deterministic algorithm was introduced by R. Carden and the connection of our problem and that of iterative eigensolvers such as Arnoldi's method and the distribution of Ritz values and Ritz pairs inside the field of values of A was made.
In the most recent work on this problem, we have developed a fast geometry based algorithm to find a unit generating vector $x \in C^n$ for a given point μ in the complex plane if this point lies inside the numerical range of A, and if not, the method determines that the given point lies outside the numerical range. It uses
(a) eigenanalyses of associated hermitian matrices
$$A(\theta) = \cos(\theta)(A + A^*)/2 \ + \ i \cdot \sin(\theta)(A - A^*)/2,$$
(b) the fact that the image of a great circle of the unit sphere in C^n under the map $x \in C^n \mapsto x^*Ax \in C$ is an ellipse inside A's numerical range,
as well as
(c) a somewhat obscure result from Horn and Johnson's Topics book for finding a unit generating vector of $0 \in C$ from known generating vectors for field of values points on the real axis that lie on both sides of 0.
The new geometric method gives very accurate results very quickly, even at close proximity of μ to A's numerical range boundary. This is independent of whether μ lies inside or out. And the MATLAB code is very short ($<$ 100 lines).

Title On the Gauss-Lucas theorem and the numerical range of companion matrices

Speaker Iwona Wróbel, Warsaw University of Technology and Institute of Mathematics of the Polish Academy of Sciences, (wrubelki@wp.pl).

Abstract

It is known that the convex hull of the roots of a given polynomial contains the roots of its derivative. This result is known as the Gauss-Lucas theorem. We will investigate the possibility of generalizing it to the numerical range of companion matrices and discuss the relation between the numerical ranges of companion matrices of a polynomial and its derivative. Several types of companion matrices will be considered.

Title Ky Fan: Beautiful Results and Beautiful Life

Speaker Fuzhen Zhang, Nova Southeastern University, Fort Lauderdale, Florida, USA, zhang@nova.edu

Abstract

This presentation is dedicated to late mathematician Ky Fan. We will take a look at some of Ky Fan's elegant inequalities on matrix majorization and present new developments on the topic.

Title On Restricted Numerical Range

Speaker Karol Zyczkowski, Jagiellonian University, Cracow, Poland

Co-authors M.-D. Choi, P.Gawron, J.A. Miszczak, Z. Puchała, Ł. Skowronek

Abstract

Let Ω be the set of complex density matrices of size N, which contains hermitian and positive operators, which are normalized by the trace condition, $\mathrm{Tr}\rho = 1$. For any operator X acting on a N-dimensional Hilbert space one defines its numerical range (field of values) as the set of all possible expectation values among normalized density operators,

$$W(X) = \{\mathrm{Tr}(X\rho) : \rho \in \Omega\}.$$

In analogy with the standard definition, for any subset $\Omega_R \in \Omega$ one defines the restricted numerical range of an operator, $W_R(X) = \{\mathrm{Tr}(X\rho) : \rho \in \Omega_R\}$. For instance one can consider the subset of real density operators. If the dimension is a composite number, $N = KM$, one defines the set of product states and the set of separable states. These sets lead to definitions of product (separable) numerical range. We review basic properties of numerical range for normal and non-normal operators and present a list of open problems.

Participant lists

* Abdolaziz Abdollahi, Shiraz University. <abdolahi@shirazu.ac.ir>
* Natalia Bebiano, University of Coimbra. <bebiano@ci.uc.pt>
* J.C. Bourin, Universite de Franche-Comte. <bourinjc@club-internet.fr>
* Man-Duen Choi, University of Toronto. <choi@math.toronto.edu>
* Dariusz Cichon, Jagiellonian University. <Dariusz.Cichon@im.uj.edu.pl>
* Michel Crouzeix, Universit\'e de Rennes 1. <michel.crouzeix@univ-rennes1.fr>
* Mirko Dobovisek, University of Ljubljana. <mirko.dobovisek@fmf.uni-lj.si>
* Roman Drnovsek, University of Ljubljana. <Roman.Drnovsek@fmf.uni-lj.si>
* Miroslav Fiedler, Academy of Sciences of the Czech Republic. <miroslav.fiedler@seznam.cz>
* Karl-Heinz Forster, Institut fur Mathematik, Technische Universitat Berlin. <foerster@math.tu-berlin.de>
* Shmuel Friedland, University of Illinois - Chicago <friedlan@uic.edu>
* Moshe Goldberg, Technion. <goldberg@math.technion.ac.il>
* Zenon Jabonski, Jagiellonian University
* Derek Kitson, Trinity College Dublin. <dk@maths.tcd.ie>
* Marek Kosied, Instytut Matematyki UJ Lojasiewicza 6. <Marek.Kosiek@im.uj.edu.pl>
* Anna Kula, Jagiellonian University. <Anna.Kula@im.uj.edu.pl>,
* Tom Laffey, University College Dublin. <tlaffey@maths.ucd.ie>
* Grzegorz Lewicki, Jagiellonian University. <Grzegorz.Lewicki@im.uj.edu.pl>
* Chi-Kwong Li, College of William and Mary (organizer). <ckli@math.wm.edu>
* Reagan Lohandjola L'okaso, University of Kinshasa. <realdemadrid002@yahoo.fr>
* Yuri Lyubich, Technion. <lyubich@techunix.technion.ac.il>
* Alexander Markus, Ben-Gurion University. <markus@cs.bgu.ac.il>
* John Maroulas, National Technical University of Athens. <maroulas@math.ntua.gr>
* Martin Mathieu, Queen's University Belfast. <m.m@qub.ac.uk>
* Javier Meri, Universidad de Granada. <jmeri@ugr.es>
* Hiroshi Nakazato, Hirosaki University. <nakahr@cc.hirosaki-u.ac.jp>
* A. Nata, CMUC and Polytechnic Institute of Tomar. <anata@ipt.pt>
* Piotr Niemiec, Jagiellonian University. <Piotr.Niemiec@im.uj.edu.pl>
* Yiu-Tung Poon, Iowa State University. <ytpoon3@gmail.com>
* Panagiotis Psarrakos, National Technical University of Athens. <ppsarr@math.ntua.gr>
* Zbigniew Puchala, Polish Academy of Sciences. <z.puchala@iitis.gliwice.pl>
* Saburou Saitoh, Gunma University. <saburou.saitoh@gmail.com>
* Abbas Salemi, University of Kerman. <asalemip@yahoo.com>
* Takashi SANO, Yamagata university, Japan. <sano@sci.kj.yamagata-u.ac.jp>
* Thomas Schulte-Herbrggen, Technical University Munich. <tosh@ch.tum.de>
* Franciszek Hugon Szafraniec, Uniwersytet Jagiellonski (organizer). <umszafra@cyf-kr.edu.pl>

* Zoltan SEBESTYN, Eotvos Univ. Budapest, Hungary. <dr.sebestyen.zoltan@gmail.com>
* Nung-Sing Sze, University of Conneticut. <Raymond.Sze@inet.polyu.edu.hk>
* Jan Stochel, Institute of Mathematics of Jagiellonian University. <Jan.Stochel@im.uj.edu.pl>
* Jerzy Bartlomiej Stochel, AGH-University of Science and Technology. <stochel@agh.edu.pl>
* Tin-Yau Tam, Auburn University. <tamtiny@auburn.edu>
* Ramazan TURKMEN, Selcuk University. <rturkmen@selcuk.edu.tr>
* Frank Ulhig, Auburn University. <uhligfd@auburn.edu>
* Iwona Wrobel, Warsaw University of Technology and Institute of Mathematics of the Polish Academy of Sciences. <i.wrobel@mini.pw.edu.pl>
* Janusz Wysoczanski, University of Wroclaw. <jwys@math.uni.wroc.pl>
* Jaroslav Zemanek, Instytut Matematyczny PAN (organizer). <zemanek@impan.gov.pl>
* Fuzhen Zhang, Nova Southeastern University. <zhang@nova.edu>
* Karol Zyczkowski, Uniwersytet Jagiellonski. <karol@tatry.if.uj.edu.pl>