Game, strategy and puzzle discussion problems

The following paragraph is copied from Problem-Soling Strategies by Arthur Engel: Most of the game problems can be solved by a simple strategy:

Divide the set of all positions into pairs, so that there is a move from the first to the second element of the pair. Whenever my opponent occupies one element of a pair, I move to the other element of the pair. Thus, I win, since my opponent runs out of moves first.

1. (Stanford training) A pile of matches are given with two players alternate playing. Each play involves removing a certain number of matches from one of the piles. The last person to play wins. Each player can remove between one and three matches. The initial pile has 10 matches. How should the first player play to win?

2. (Stanford training) Solve the mis‘ere version of the previous game: the last person to play loses.

3. (Stanford training) A pile of matches are given with two players alternate playing. Each play involves removing a certain number of matches from one of the piles. The last person to play wins. Each player can remove 2^n matches, for any non-negative integer n. How should the first player play to win?

4. (Stanford training) Same situation as before. There are four piles of matches, with 7, 8, 9, and 10 matches respectively. Each player can remove between one and three matches from one of the piles. How should the first player play to win?

5. (Putnam 2002-A4) In Determinant Tic-Tac-Toe, Player 1 enters a 1 in an empty 3×3 matrix. Player 0 counters with a 0 in a vacant position, and play continues in turn until the 3×3 matrix is completed with five 1's and four 0's. Player 0 wins if the determinant is 0 and player 1 wins otherwise. Assuming both players pursue optimal strategies, who will win and how?

6. (PCMI training) Im going to give each of you a hat to wear that is either black or white. You cannot see a hat on your head, but you can see a hat on anyone elses head. Your Goal as a group is to organize yourselves in a line, with all the white-hatted folks on one end, and all the black-hatted folks on the other end. Note that after I give you your hats, you cannot communicate with (nor touch) anyone. However, before I give you the hats you may jointly decide on a strategy. (a) Devise a strategy that will achieve the Goal. (b) Can you find a symmetric strategy, *i.e.*, one that is the same for all players?

7. (MIT training 2.5 star) Persons X and Y have nonnegative integers painted on their foreheads which only the other can see. They are told that the sum of the two numbers is either 100 or 101. A third person P asks X if he knows the number on his forehead. If X says "no", then P asks Y. If Y says "no", then P asks X again, etc. Assume both X and Y are perfect logicians. Show that eventually one of them will answer "yes". (This may seem paradoxical. For instance, if X and Y both have 50 then Y knows that X will answer "no" to the first question, since from Y's viewpoint X will see either 50 or 51, and in either case cannot deduce his number. So how does either person gain information?)