Presentation problems for congruence

1. Show that $4^{3x+1} + 2^{3x+1} + 1$ is divisible by 7.
2. Prove that the sum of two odd squares cannot be a square.
3. What are the last two digits of 3^{1234}?
   (Hint: show $3^{1234} \equiv k(mod 100)$ for some $0 \leq k \leq 99$.)
4. Prove that if $2n + 1$ and $3n + 1$ are both perfect squares, then n is divisible by 40. (Hint: show the are both divisible by 5 and 8.)
5. Show that $222 \cdots 222$ (1980 2's) is divisible by 1982. (Hint: Fermat's Little Theorem)
6. (Sun Tzu, about 473 AD to 280 AD) We have a number of things, but we do not know exactly how many. If we count them by threes we have two left over. If we count them by fives we have three left over. If we count them by sevens we have two left over. How many things are there? (Hint: Chinese Remainder Theorem)
7. (Putnam 1955) Do there exist 1,000,000 consecutive integers each of which contains a repeated prime factor? (Hint: Chinese Remainder Theorem)